

Proposition $I(\mathbf{X}'; \mathbf{Y}') = I(\mathbf{X}; \mathbf{Y})$.

Proof

$$\begin{aligned}
I(\mathbf{X}'; \mathbf{Y}') &= h(\mathbf{Y}') - h(\mathbf{Y}' | \mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}' | \mathbf{X}') \\
&= h(\mathbf{Y}') - h(\mathbf{Z}') \\
&= h(Q^\top \mathbf{Y}) - h(Q^\top \mathbf{Z}) \\
&= \left[h(\mathbf{Y}) + \log |\det(Q^\top)| \right] \\
&\quad - \left[h(\mathbf{Z}) + \log |\det(Q^\top)| \right] \\
&= h(\mathbf{Y}) - h(\mathbf{Z}) \\
&= h(\mathbf{Y}) - h(\mathbf{Z} | \mathbf{X}) \\
&= h(\mathbf{Y}) - h(\mathbf{Y} | \mathbf{X}) \\
&= I(\mathbf{X}; \mathbf{Y}).
\end{aligned}$$

- Therefore, the equivalent system and the original system have the same capacity.
- Hence, the capacity of a system of correlated Gaussian channels is given by

$$\frac{1}{2} \sum_{i=1}^k \log \left(1 + \frac{a_i^*}{\lambda_i} \right),$$

where a_i^* is the optimal power allocated to the i th channel in the equivalent system, and its value can be obtained by water-filling.