Maximize $\sum_i \log(P_i + N_i)$ subject to $\sum_i P_i \leq P$ and $P_i \geq 0$.

Lagrange Multipliers:

1. Apply the method of Lagrange multipliers by temporarily ignoring the nonnegativity constraints on P_i.

2. Observe that in order for $\sum_i \log(P_i + N_i)$ to be maximized, $\sum_i P_i = P$ must hold because $\log(P_i + N_i)$ is increasing in P_i.

3. Therefore, set $\sum_i P_i = P$.

4. Let

$$J = \sum_{i=1}^{k} \log(P_i + N_i) - \mu \sum_{i=1}^{k} P_i.$$

5. Differentiating with respect to P_i gives

$$\frac{\partial J}{\partial P_i} = \frac{\log e}{P_i + N_i} - \mu.$$

6. Setting $\frac{\partial J}{\partial P_i} = 0$, we have

$$P_i = \frac{\log e}{\mu} - N_i.$$

7. Upon letting $\nu = \frac{\log e}{\mu}$, we have

$$P_i = \nu - N_i, \tag{1}$$

where ν is chosen to satisfy the power constraint

$$\sum_{i=1}^{k} P_i = \sum_{i=1}^{k} (\nu - N_i) = P.$$

This solution has a water-filling interpretation.

8. Note that $P_i \geq 0$ if and only if $\nu \geq N_i$. Thus $P_i \geq 0$ for all i if and only if $\nu \geq N_i$ for all i. However, this is not guaranteed.

9. Nevertheless, (1) suggests the general solution to be proved in Proposition 11.23.