Maximization of $\sum_i \log(P_i + N_i)$

- Constraints: $\sum_i P_i \leq P$ and $P_i \geq 0$
- $\sum_i P_i \leq P$ can be replaced by $\sum_i P_i = P$ because $\log(P_i + N_i)$ is increasing in P_i.
- Ignore the constraints $P_i \geq 0$ for the time being. Use Lagrange multiplier to obtain

$$P_i = \nu - N_i$$

  where the constant ν is chosen such that

$$\sum_{i=1}^{k} P_i = \sum_{i=1}^{k} (\nu - N_i) = P$$

- This solution, which has a water-filling interpretation, would be a valid solution if $\nu \geq N_i$ so that $P_i \geq 0$ for all i.