Theorem 11.21 (Capacity of a Memoryless Gaussian Channel) The capacity of a memoryless Gaussian channel with noise power N and input power constraint P is

$$\frac{1}{2} \log \left(1 + \frac{P}{N} \right).$$

The capacity is achieved by the input distribution $\mathcal{N}(0, P)$.

Remarks

- The capacity of a memoryless Gaussian channel depends only on P/N, called the signal-to-noise ratio.

- The capacity is strictly positive no matter how small P/N is.

- The capacity is infinite if there is no input power constraint.

Lemma 11.22 Let $Y = X + Z$. Then $h(Y|X) = h(Z|X)$ provided that $f_{Z|X}(z|x)$ exists for all $x \in \mathcal{S}_X$.