Proof

1. Assume that $f_{Z|X}(z|x)$ exists for all $x \in \mathcal{S}_X$.

2. Consider

$$
\begin{aligned}
f_{Y|X}(y|x) &= f_{X+Z|X}(y|x) \\
&= f_{x+Z|X}(y|x) \\
&= f_{(x+Z)-x|X}(y-x|x) \\
&= f_{Z|X}(y-x|x).
\end{aligned}
$$

Thus $f_{Y|X}(y|x)$ exists.

3. Then $h(Y|X=x)$ is defined, and

$$
\begin{aligned}
h(Y|X) &= \int h(Y|X=x)dF_X(x) \\
&= \int h(X+Z|X=x)dF_X(x) \\
&= \int h(x+Z|X=x)dF_X(x) \\
&= \int h(Z|X=x)dF_X(x) \\
&= h(Z|X).
\end{aligned}
$$