Parameter Settings

1. Fix $\epsilon > 0$ and input distribution $F(x)$. Let δ to be specified later.

2. Since $C(P)$ is left-continuous, there exists $\gamma > 0$ such that

$$C(P - \gamma) > C(P) - \frac{\epsilon}{6}.$$

3. By the definition of $C(P-\gamma)$, there exists an input random variable X such that

$$E\kappa(X) \le P - \gamma \quad \text{and} \quad I(X;Y) \ge C(P-\gamma) - \frac{\epsilon}{6}.$$

4. Choose for a sufficiently large n an even integer M satisfying

$$I(X;Y) - \frac{\epsilon}{6} < \frac{1}{n}\log M < I(X;Y) - \frac{\epsilon}{8}.$$

5. Then

$$\frac{1}{n}\log M > I(X;Y) - \frac{\epsilon}{6} \ge C(P-\gamma) - \frac{\epsilon}{3} > C(P) - \frac{\epsilon}{2}.$$

The Random Coding Scheme

1. Construct the codebook $\mathcal{C}$ of an (n, M) code randomly by generating M codewords in $\Re^n$ independently and identically according to $F(x)^n$.

2. Denote these codewords by $\tilde{\mathbf{X}}(1), \tilde{\mathbf{X}}(2), \cdots, \tilde{\mathbf{X}}(M)$.

3. Reveal the codebook $\mathcal{C}$ to both the encoder and the decoder.

4. A message W is chosen from $\mathcal{W}$ uniformly.

5. The sequence $\mathbf{X} = \tilde{\mathbf{X}}(W)$ is transmitted through the channel.

6. The channel outputs a sequence $\mathbf{Y}$ according to

$$\Pr\{Y_i \le y_i, 1 \le i \le n | \mathbf{X}(W) = \mathbf{x}\} = \prod_{i=1}^{n} \int_{-\infty}^{y_i} f(y|x_i)dy.$$

7. The sequence $\mathbf{Y}$ is decoded to the message w if

- $(\mathbf{X}(w), \mathbf{Y}) \in \Psi^n_{[XY]\delta}$, and
- there does not exist $w' \neq w$ such that $(\mathbf{X}(w'), \mathbf{Y}) \in \Psi^n_{[XY]\delta}$.

Otherwise, $\mathbf{Y}$ is decoded to a constant message in $\mathcal{W}$. Denote by $\hat{W}$ the message to which $\mathbf{Y}$ is decoded.