

5. By WLLN, for sufficiently large n ,

$$\Pr\{E_e|W = 1\}$$

$$= \Pr\left\{\frac{1}{n} \sum_{i=1}^n \kappa(\tilde{X}_i(1)) > P \middle| W = 1\right\}$$

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$$= \Pr\left\{\frac{1}{n} \sum_{i=1}^n \kappa(\tilde{X}_i(1)) > (P - \gamma) + \gamma\right\}$$

$$\leq \Pr\left\{\frac{1}{n} \sum_{i=1}^n \kappa(\tilde{X}_i(1)) > E\kappa(X) + \gamma\right\}$$

$$\leq \frac{\epsilon}{4}.$$

6. Therefore,

$$\Pr\{Err\} \leq \frac{\epsilon}{2}$$

which implies for some codebook \mathcal{C}^* ,

$$\Pr\{Err|\mathcal{C}^*\} \leq \frac{\epsilon}{2}.$$

7. Rank the codewords in \mathcal{C}^* in ascending order according to $\Pr\{Err|\mathcal{C}^*, W = w\}$.

8. After discarding the worst half of the codewords in \mathcal{C}^* , if a codeword $\tilde{\mathbf{X}}(w)$ remains in \mathcal{C}^* , then

$$\Pr\{Err|\mathcal{C}^*, W = w\} \leq \epsilon.$$

9. However, it is not clear whether $\tilde{\mathbf{X}}(w)$ satisfies both $\lambda_w \leq \epsilon$ and the input constraint.

10. Since $Err = E_e \cup E_d$, we have

$$\lambda_w = \Pr\{E_d|\mathcal{C}^*, W = w\} \leq \epsilon$$

and

$$\Pr\{E_e|\mathcal{C}^*, W = w\} \leq \epsilon.$$

11. Observe that conditioning on $\{\mathcal{C}^*, W = w\}$, the codeword $\tilde{\mathbf{X}}(w)$ is deterministic, so either

$$\Pr\{E_e|\mathcal{C}^*, W = w\} = 0$$

or

$$\Pr\{E_e|\mathcal{C}^*, W = w\} = 1.$$

Therefore, $\Pr\{E_e|\mathcal{C}^*, W = w\} = 0$.