1. Let $\tilde{\mathbf{X}}(w) = (\tilde{X}_1(w), \tilde{X}_2(w), \cdots, \tilde{X}_n(w))$.

2. Define the error event $Err = E_e \cup E_d$, where

$$E_e = \left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(W)) > P \right\}$$

and

$$E_d = \{\hat{W} \neq W\}.$$

3. Consider

$$\begin{aligned}\Pr\{Err\} &= \Pr\{Err|W = 1\} \\ &\leq \Pr\{E_e|W = 1\} + \Pr\{E_d|W = 1\}.\end{aligned}$$

4. The analysis of $\Pr\{E_d|W = 1\}$ is exactly the same as the analysis of the decoding error in the discrete case. So we can choose δ to be small to make

$$\Pr\{E_d|W = 1\} \leq \frac{\epsilon}{4}$$

for sufficiently large n.

5. By WLLN, for sufficiently large n,

$$\begin{aligned}&\Pr\{E_e|W = 1\} \\ &= \Pr\left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > P \,\middle|\, W = 1 \right\} \\ &= \Pr\left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > P \right\} \\ &= \Pr\left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > (P - \gamma) + \gamma \right\} \\ &\leq \Pr\left\{ \frac{1}{n} \sum_{i=1}^{n} \kappa(\tilde{X}_i(1)) > E\kappa(X) + \gamma \right\} \\ &\leq \frac{\epsilon}{4}.\end{aligned}$$

6. Therefore,

$$\Pr\{Err\} \leq \frac{\epsilon}{2}$$

which implies for some codebook $\mathcal{C}^*$,

$$\Pr\{Err|\mathcal{C}^*\} \leq \frac{\epsilon}{2}.$$