

1. Consider

$$\frac{1}{n} \log \frac{f(\mathbf{Y}|\mathbf{X})}{f(\mathbf{Y})} = \frac{1}{n} \log \prod_{i=1}^n \frac{f(Y_i|X_i)}{f(Y_i)} = \frac{1}{n} \sum_{i=1}^n \log \frac{f(Y_i|X_i)}{f(Y_i)}.$$

2. By WLLN,

$$\frac{1}{n} \sum_{i=1}^n \log \frac{f(Y_i|X_i)}{f(Y_i)} \rightarrow E \log \frac{f(Y|X)}{f(Y)} = I(X; Y)$$

in probability.