- Then for small Δ,

$$\begin{aligned}
H(\hat{X}_{\Delta}) &= -\sum_i p_i \log p_i \\
&\approx -\sum_i (f(x_i)\Delta) \log(f(x_i)\Delta) \\
&\approx -\sum_i (f(x_i)\Delta) \left(\log f(x_i) + \log \Delta\right) \\
&= -\sum_i f(x_i)\Delta \log f(x_i) - \sum_i f(x_i)\Delta \log \Delta \\
&= -\sum_i [f(x_i) \log f(x_i)]\Delta - (\log \Delta) \sum_i f(x_i)\Delta \\
&\approx -\int f(x) \log f(x) dx - (\log \Delta) \int f(x) dx \\
&= h(X) - \log \Delta.
\end{aligned}$$

- Note that $H(\hat{X}_{\Delta}) \to \infty$ as $\Delta \to 0$.