

- A real r.v.  $X$  with cumulative distribution function  $F_X(x) = \Pr\{X \leq x\}$  (CDF) is
  - **discrete** if  $F_X(x)$  increases only at a countable number of values of  $x$ ;
  - **continuous** if  $F_X(x)$  is continuous, or equivalently,  $\Pr\{X = x\} = 0$  for every value of  $x$ ;
  - **mixed** if  $F_X(x)$  is neither discrete nor continuous.
- $\mathcal{S}_X$  is the set of all  $x$  such that  $F_X(x) > F_X(x - \epsilon)$  for all  $\epsilon > 0$ .

$$Eg(X) = \int_{\mathcal{S}_X} g(x) dF_X(x),$$

where the right hand side is a Lebesgue-Stieltjes integration which covers all cases (i.e., discrete, continuous, and mixed) for the CDF  $F_X(x)$ .