

Let \mathbf{q}_i be the i th column of Q . Then

$$\mathbf{q}_i \mathbf{q}_j^\top = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j. \end{cases}$$

Therefore,

$$QQ^\top = \left[\begin{array}{c|ccc} & & & \\ \mathbf{q}_1 & & & \\ & & & \\ & \cdots & & \\ & & \mathbf{q}_n & \\ & & & \end{array} \right] \left[\begin{array}{ccc} \text{---} & \mathbf{q}_1^\top & \text{---} \\ & \vdots & \\ \text{---} & \mathbf{q}_n^\top & \text{---} \end{array} \right] = [\mathbf{q}_i \mathbf{q}_j^\top] = I.$$

Hence, $Q^{-1} = Q^\top$.