

**Lemma 10.6** Let  $\mathbf{X}$  and  $\mathbf{Y}$  be column vectors of  $n$  random variables such that

$$\mathbf{Y} = Q^\top \mathbf{X},$$

where  $Q\Lambda Q^\top$  is a diagonalization of  $K_{\mathbf{X}}$ . Then  $K_{\mathbf{Y}} = \Lambda$ , i.e., the random variables in  $\mathbf{Y}$  are uncorrelated and  $\text{var } Y_i = \lambda_i$ , the  $i$ th diagonal element of  $\Lambda$ .

### Proof

1. By Proposition 10.5,

$$\begin{aligned} K_{\mathbf{Y}} &= Q^\top K_{\mathbf{X}} Q \\ &= Q^\top (Q\Lambda Q^\top) Q \\ &= (Q^\top Q)\Lambda(Q^\top Q) \\ &= \Lambda. \end{aligned}$$

2. Since  $K_{\mathbf{Y}} = \Lambda$  is a diagonal matrix, the random variables in  $\mathbf{Y}$  are uncorrelated because

$$\text{cov}(Y_i, Y_j) = 0$$

for  $i \neq j$ .

3. Furthermore, the variance of  $Y_i$  is given by the  $i$ th diagonal element of  $K_{\mathbf{Y}} = \Lambda$ , i.e.,  $\lambda_i$ . The proposition is proved.