

Proposition 10.4 The eigenvalues of a positive semidefinite matrix are non-negative.

Proof

1. Consider eigenvector $\mathbf{q} \neq 0$ and corresponding eigenvalue λ of K , i.e.,

$$K\mathbf{q} = \lambda\mathbf{q}.$$

2. Since K is positive semidefinite,

$$0 \leq \mathbf{q}^\top K\mathbf{q} = \mathbf{q}^\top (\lambda\mathbf{q}) = \lambda(\mathbf{q}^\top \mathbf{q}).$$

3. $\lambda \geq 0$ because $\mathbf{q}^\top \mathbf{q} = \|\mathbf{q}\|^2 > 0$.

Remark Since a covariance matrix is both symmetric and positive semidefinite, it is diagonalizable and its eigenvalues are nonnegative.