

Proposition 10.5 Let $\mathbf{Y} = A\mathbf{X}$. Then

$$K_{\mathbf{Y}} = AK_{\mathbf{X}}A^{\top}$$

and

$$\tilde{K}_{\mathbf{Y}} = A\tilde{K}_{\mathbf{X}}A^{\top}.$$

Proposition 10.6 (Decorrelation) Let $\mathbf{Y} = Q^{\top}\mathbf{X}$, where $K_{\mathbf{X}} = Q\Lambda Q^{\top}$. Then $K_{\mathbf{Y}} = \Lambda$, i.e.,

1. the random variables in \mathbf{Y} are uncorrelated
2. $\text{var } Y_i = \lambda_i$ for all i

Corollary 10.7 Any random vector \mathbf{X} can be written as a linear transformation of an uncorrelated vector. Specifically, $\mathbf{X} = Q\mathbf{Y}$, where $K_{\mathbf{X}} = Q\Lambda Q^{\top}$.

Proof In Proposition 10.6, $\mathbf{Y} = Q^{\top}\mathbf{X}$ implies $Q\mathbf{Y} = QQ^{\top}\mathbf{X}$, or $\mathbf{X} = Q\mathbf{Y}$.