

**Proposition 10.9** Let  $\mathbf{Y} = Q\mathbf{X}$ , where  $\mathbf{X}$  and  $\mathbf{Y}$  are column vectors of  $n$  random variables and  $Q$  is an orthogonal matrix. Then

$$E \sum_{i=1}^n Y_i^2 = E \sum_{i=1}^n X_i^2.$$

**Proof**

1. Consider

$$\begin{aligned} \sum_{i=1}^n Y_i^2 &= \mathbf{Y}^\top \mathbf{Y} \\ &= (Q\mathbf{X})^\top (Q\mathbf{X}) \\ &= (\mathbf{X}^\top Q^\top)(Q\mathbf{X}) \\ &= \mathbf{X}^\top (Q^\top Q)\mathbf{X} \\ &= \mathbf{X}^\top \mathbf{X} \\ &= \sum_{i=1}^n X_i^2. \end{aligned}$$

2. The proposition is proved upon taking expectation on both sides.