



CENG4480

Lecture 04: Analog/Digital Conversions

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Overview



Preliminaries

Comparator

Digital to Analog Conversion (DAC)

Analog to Digital Conversion (ADC)

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Analog to Digital Conversion (ADC)

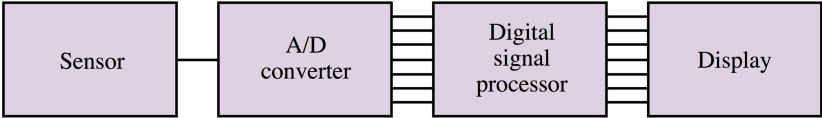
Analog/Digital Conversions



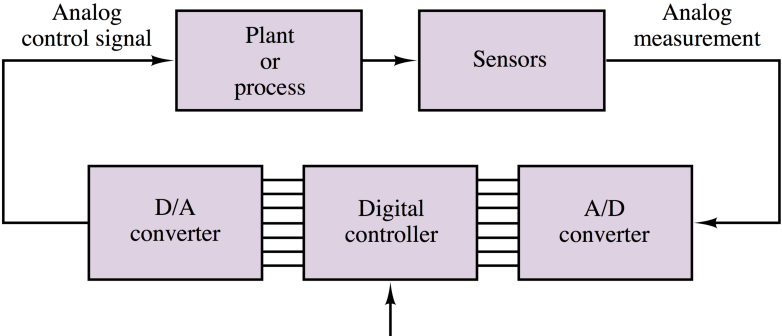
Topics:

- ▶ Digital to analog conversion
- ▶ Analog to digital conversion
- ▶ Sampling-speed limitation
- ▶ Frequency aliasing
- ▶ Practical ADCs of different speed

Block Diagrams



Digital measuring instrument



Digital control system

Overview



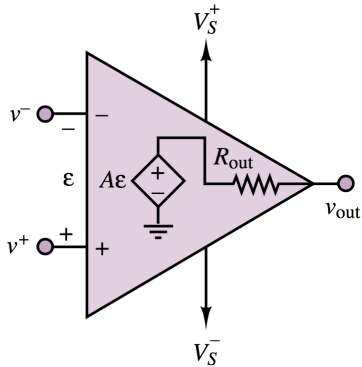
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Op-Amp Comparator

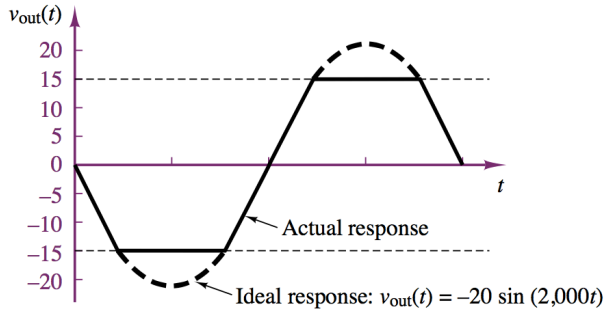


Open-Loop Mode

$$v_{out} = A_V(v^+ - v^-)$$

- ▶ Extreme large gain
- ▶ Any small difference ϵ will cause large outputs.

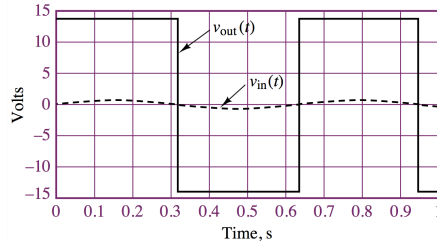
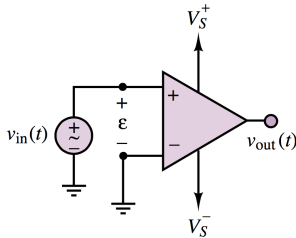
Voltage Supply Limits



Op-amp output with voltage supply limit ($V_S^+ = V_S^- = 15$)

- ▶ Powered by external DC voltage supplies V_S^+ & V_S^-
- ▶ Amplifying signals **only** within the range of supply voltages
- ▶ In a practical op-amp, **saturation** would be reached at 1.5 V below the supply voltages.

Switching waveforms by Comparator



Switching waveforms of non-inverting comparator.

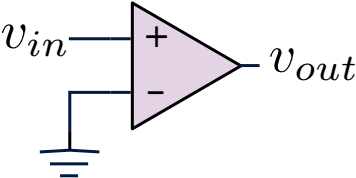
Since $\epsilon = V \cos(\omega t)$, therefore

$$\epsilon > 0 \Rightarrow v_{out} = V_{sat}^+$$

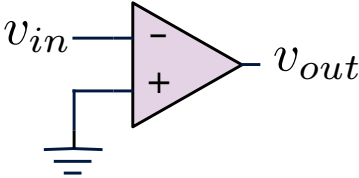
$$\epsilon < 0 \Rightarrow v_{out} = V_{sat}^-$$

* V_{sat} : saturation voltage (e.g., ± 15 V supplies is approximately ± 13.5 V)

Noninverting & Inverting Comparator

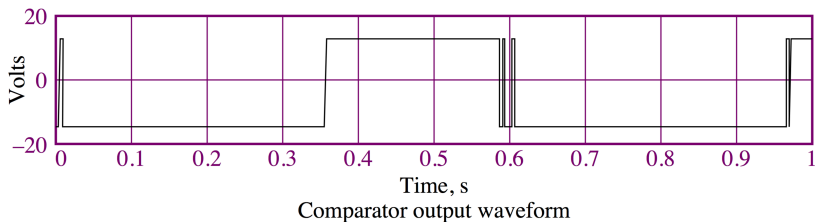
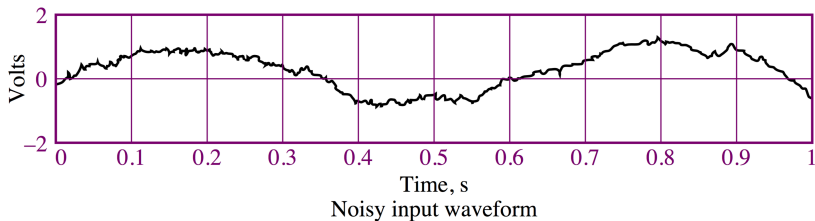


(a) Noninverting comparator



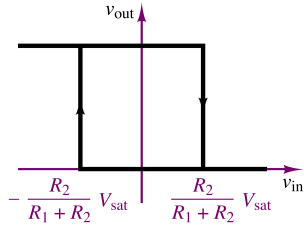
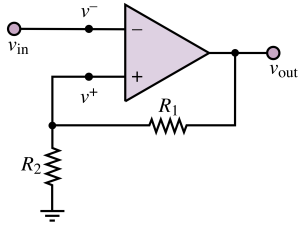
(b) Inverting comparator

Limitation of Conventional Comparator

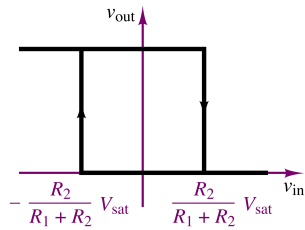
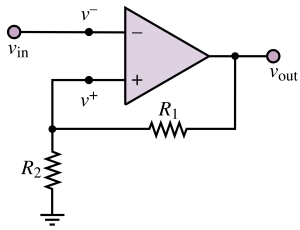


- ▶ In the presence of noisy inputs
- ▶ Cross the reference voltage level repeatedly
- ▶ Cause multiple triggering

Schmitt Trigger



- ▶ Based on **Inverting** comparator
- ▶ Positive feedback
- ▶ (+) Increase the switching speed
- ▶ (+) Noise immunity



Question: prove two reference voltages of schmitt trigger.

Sample-and-Hold Amplifier

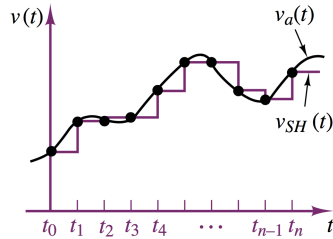
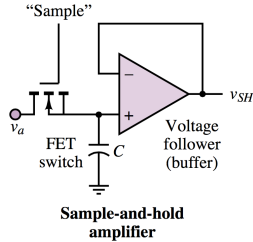


Motivations:

When a slow ADC is used to sample a fast changing signal only a short sampling point can be analyzed

- ▶ To resolve uncertainty during ADC
- ▶ “freeze” the value of analog waveform for a time sufficient for the ADC to complete its task

Sample-and-Hold Amplifier



- ▶ A MOSFET analog switch is used to “sample” analog waveform
- ▶ While MOSFET conducts, charge the “hold” capacitor

Good Sample, Bad Sample

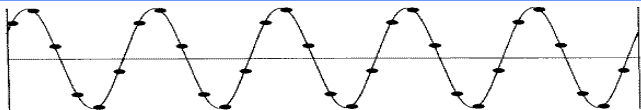


Figure 3.14 When a sine wave is sampled six times per cycle ($f_s = 6f$), the observed frequency is equal to the true frequency.

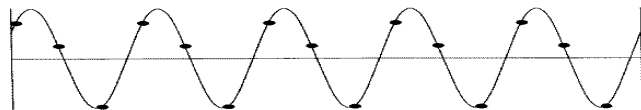


Figure 3.15 When a sine wave is sampled three times per cycle ($f_s = 3f$), the observed frequency is equal to the true frequency.



Figure 3.16 When a sine wave is sampled six times in five cycles, the observed sine wave has a much lower frequency than the original sine wave.

- ▶ When sampling 6 times per cycle, close to the original.
- ▶ when sampling 3 times per cycle, less reliable but frequency is equal to original.
- ▶ When sampling 6 times per 5 cycles, frequency is different.

Overview



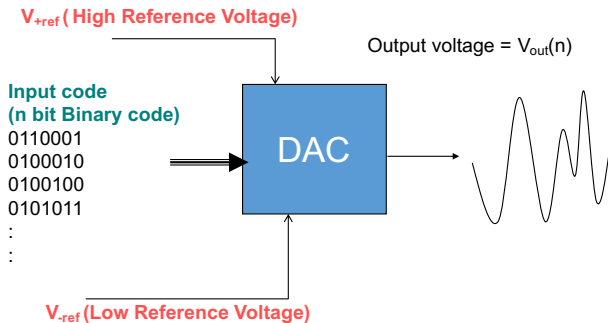
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Digital to Analog Conversion (DAC)

Analog to Digital Conversion (ADC)

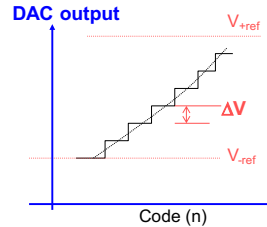
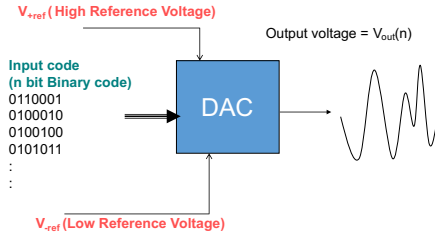
Digital-to-Analog Converter (DAC)



$$\begin{aligned}V_{out} &= (b_3b_2b_1b_0)_2 \\ &= (b_3 \cdot 2^3 + b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0)_{10} \\ &= (8b_3 + 4b_2 + 2b_1 + b_0)\Delta v + V_{-ref}\end{aligned}$$

Δv : smallest step size by which voltage can increase

How to Determine Δv ?



$$\Delta v = \frac{V_{+ref} - V_{-ref}}{2^n},$$

where n is the bit# of input digital signal.



Glitch:

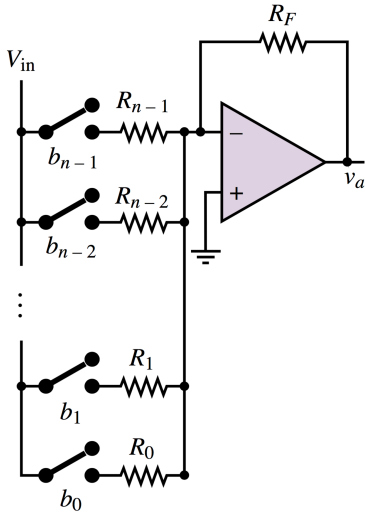
A transient spike in the output of a DAC that occurs when more than one bit changes in the input code.

- ▶ Use a low pass filter to reduce the glitch
- ▶ Use **sample-and-hold** circuit to reduce the glitch

Settling time:

Time for the output to settle to typically $1/4$ LSB after a change in DA output.

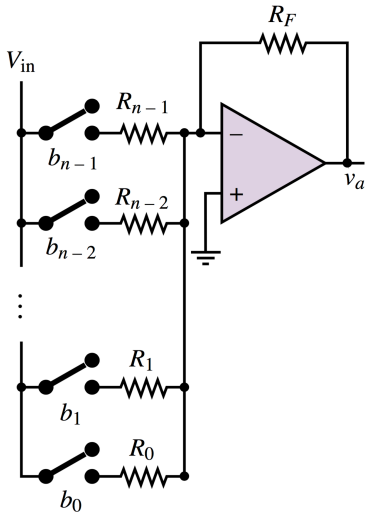
DAC Type 1: Weighted Adder DAC



Similar to [summing amplifier](#):

$$v_a = - \sum_i \left(\frac{R_F}{R_i} \cdot b_i \cdot v_{in} \right)$$

DAC Type 1: Weighted Adder DAC



Similar to [summing amplifier](#):

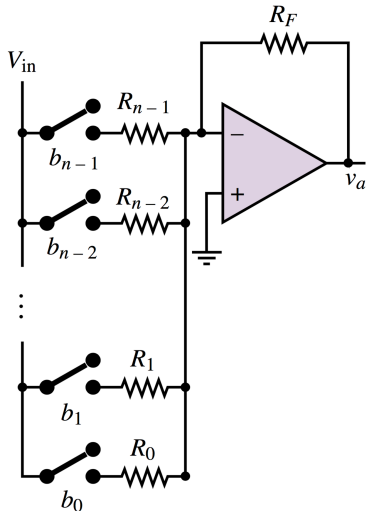
$$v_a = - \sum_i \left(\frac{R_F}{R_i} \cdot b_i \cdot v_{in} \right)$$

If we select $R_i = \frac{R_0}{2^i}$:

$$v_a = - \frac{R_F}{R_0} (2^{n-1} b_{n-1} + \dots + 2^1 b_1 + 2^0 b_0) \cdot v_{in}$$

Note here V_{-ref} is 0 (ground)

DAC Type 1: Weighted Adder DAC



Limitations:

- ▶ Impossible to fabricate a **wide range** of resistor values in the same IC chip

Similar to **summing amplifier**:

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If we select $R_i = \frac{R_0}{2^i}$:

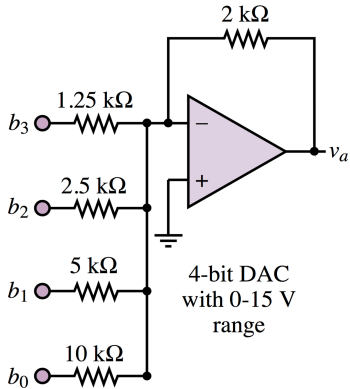
$$v_a = - \frac{R_F}{R_0} (2^{n-1} b_{n-1} + \dots + 2^1 b_1 + 2^0 b_0) \cdot v_{in}$$

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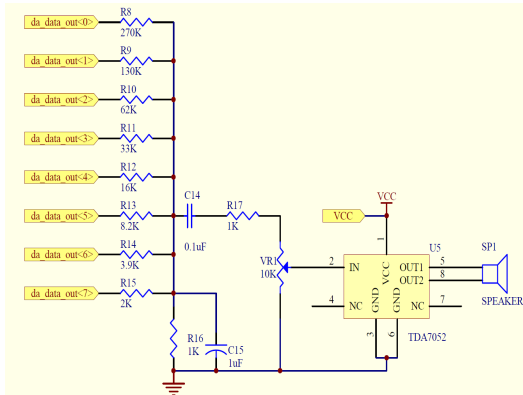


Question: 4-bit DAC

For given $(b_3b_2b_1b_0) = \{(1111), (0000), (1010)\}$, calculate v_a .



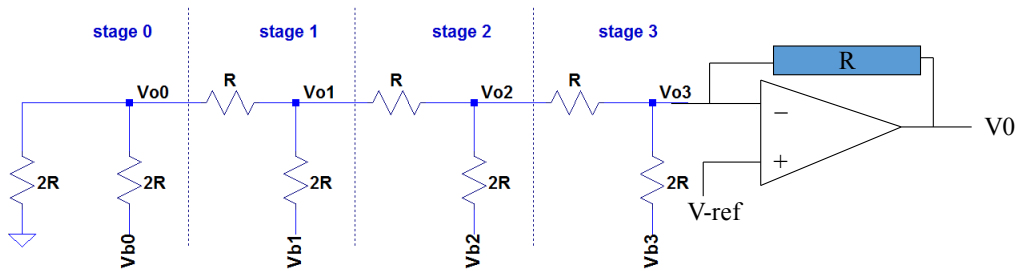
Practical Resistor Network DAC and Audio Amplifier



Data Bit	Ideal R	Real R
0 (LSB)	256K	270K
1	128K	130K
2	64K	62K
3	32K	33K
4	16K	16K
5	8K	8.2K
6	4K	3.9K
7 (MSB)	2K	2K

► Not perfect, but okay.

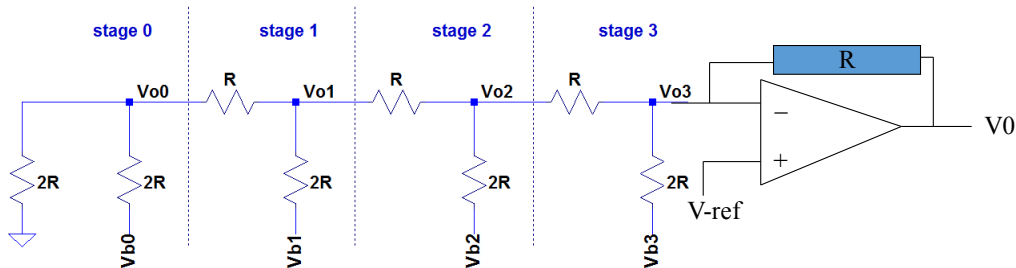
DAC Type 2: R-2R DAC



Motivations:

- ▶ Use only **two** values of resistors which make for easy and accurate fabrication and integration
- ▶ At each node, current is split into 2 equal parts
- ▶ The most popular DAC

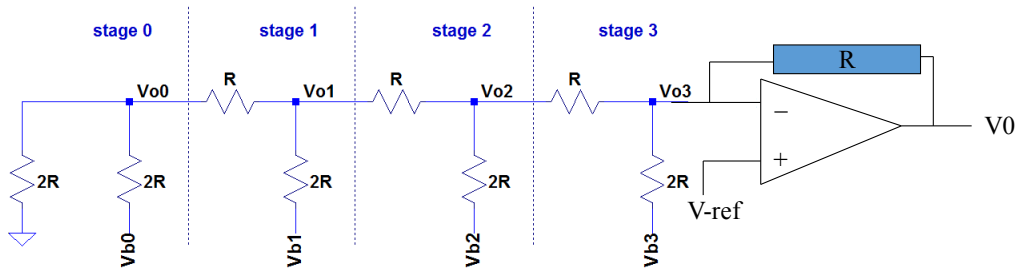
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Reference:

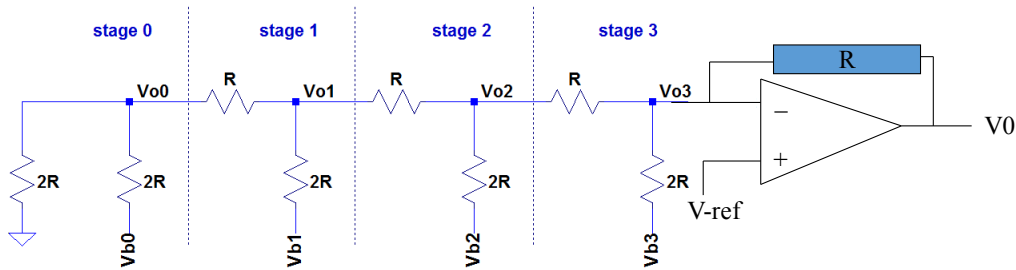
<http://www.tek.com/blog/tutorial-digital-analog-conversion—r-2r-dac>

DAC Type 2: R-2R DAC



Given I as input value (n bit):

$$V_{o3} = \frac{V_{b0}}{16} + \frac{V_{b1}}{8} + \frac{V_{b2}}{4} + \frac{V_{b3}}{2}$$



Question: R-2R DAC

For given $(b_3b_2b_1b_0) = \{(1111), (0000), (1010)\}$, calculate v_{o3} .

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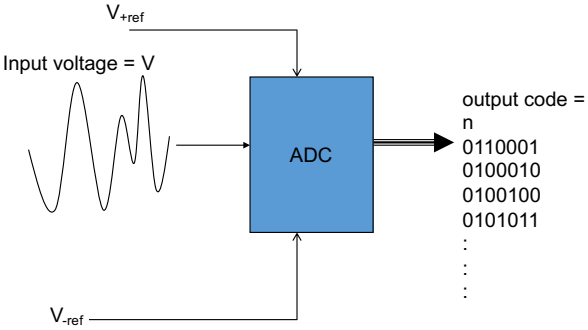
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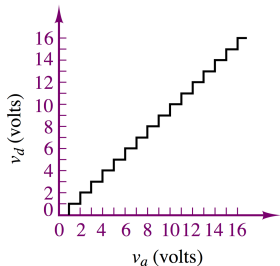
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Analog-to-Digital Converter (ADC)



Quantization

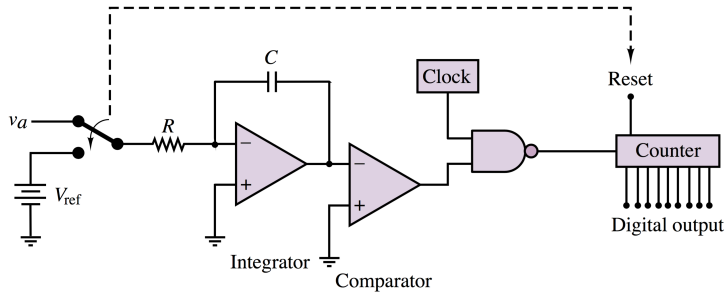


Quantized voltage Binary representation

v_d	b_3	b_2	b_1	b_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
\vdots		\vdots		
14	1	1	1	0
15	1	1	1	1

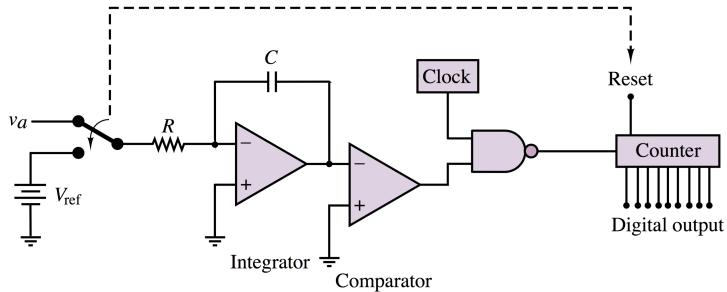
- ▶ Convert an analog level to digital output
- ▶ Employ $2^n - 1$ intervals (n : bit#)
- ▶ v_a : analog voltage
- ▶ v_d : output digital voltage

ADC Type 1: Integrating ADC



- ▶ Accumulate the input current on a capacitor for a fixed time
- ▶ Then measure time (T) to discharge the capacitor
- ▶ When cap is discharged to 0 V, comparator will stop the counter

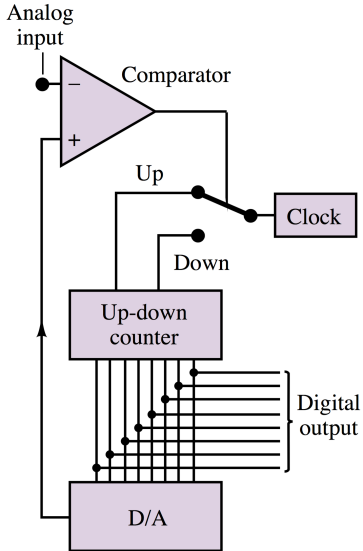
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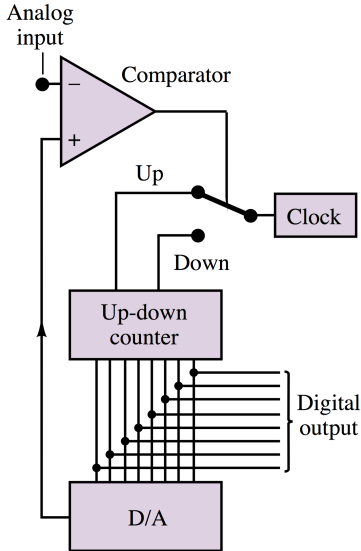
Limination: Slow

ADC Type 2: Tracking ADC



- ▶ ADC repeatedly compares its input with DAC outputs
- ▶ Up/down count depends on input/DAC output comparison

ADC Type 2: Tracking ADC



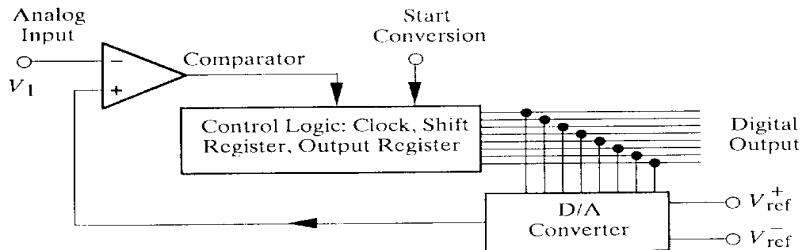
- ▶ ADC repeatedly compares its input with DAC outputs
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Limination: Slow

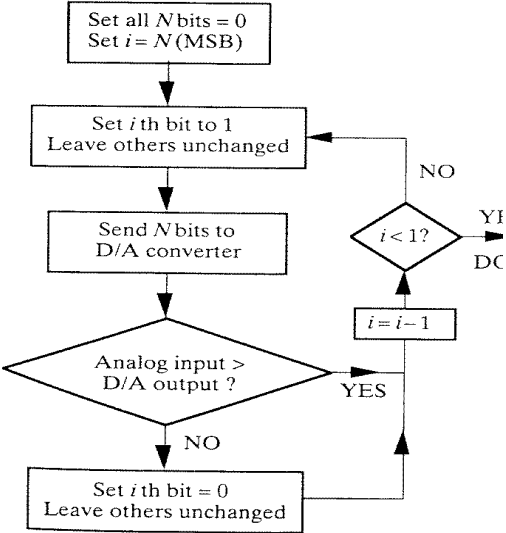
ADC Type 3: Successive Approximation



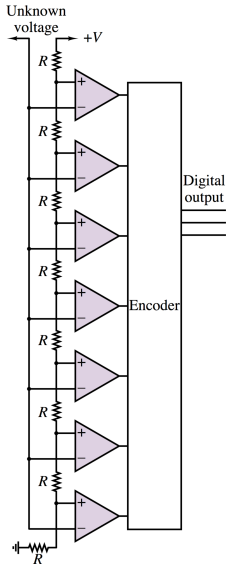
- ▶ Replace “Up-down counter” by “control logic”
- ▶ Binary search to determine the output bits
- ▶ still slow although faster than types 1 & 2



Flow chart of Successive-approximation ADC



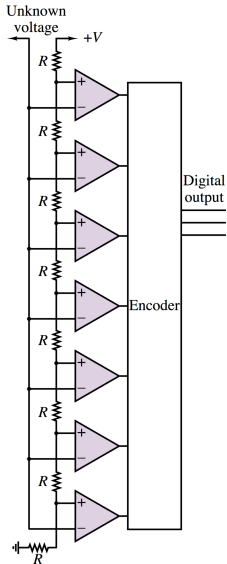
ADC Type 4: Flash ADC



- ▶ Divide the voltage range into $2^n - 1$ levels
- ▶ Use $2^n - 1$ comparators to determine what the voltage level is
- ▶ Fully parallel

Pros:

ADC Type 4: Flash ADC



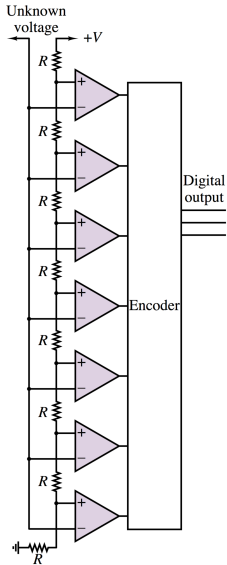
- ▶ Divide the voltage range into $2^n - 1$ levels
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Pros:

- ▶ Very fast for high quality audio and video
- ▶ **Sample and hold** circuit **NOT** required

Cons:

ADC Type 4: Flash ADC



- ▶ Divide the voltage range into $2^n - 1$ levels
- ▶ Use $2^n - 1$ comparators to determine what the voltage level is
- ▶ Fully parallel

Pros:

- ▶ Very fast for high quality audio and video
- ▶ **Sample and hold** circuit **NOT** required

Cons:

- ▶ Very expensive for wide bits conversion