



CENG4480

## Lecture 03: Operational Amplifier – 2

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# Overview



Preliminaries

Integrator & Differentiator

Filters

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# Euler's Identity



$$e^{j\theta} = \cos\theta + j\sin\theta$$

- ▶ real component
- ▶ imaginary component
- ▶ magnitude

$$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$



Prove:

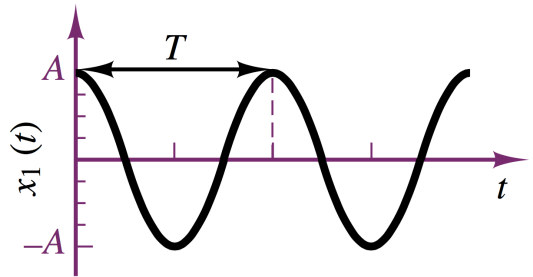
$$\left| \frac{1}{1 + ja} \right| = \frac{1}{\sqrt{1 + a^2}}$$

# Sinusoidal Signal



$$x(t) = A\cos(\omega t + \phi)$$

- ▶ Periodic signals
- ▶  $A$ : amplitude
- ▶  $\omega$ : radian frequency
- ▶  $\phi$ : phase

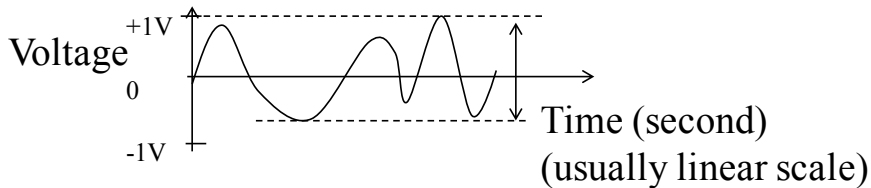




- ▶ Voltage gain against time

For sinusoidal signal:

$$v(t) = A\cos(\omega t + \phi)$$

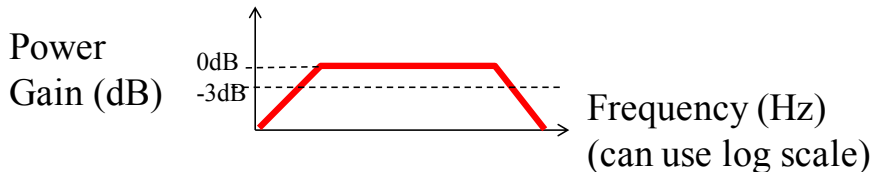




- ▶ Voltage gain against frequency

For sinusoidal signal:

$$\mathbf{V}(j\omega) = Ae^{j\phi} = A\angle\phi = A\cos\phi + jA\sin\phi$$

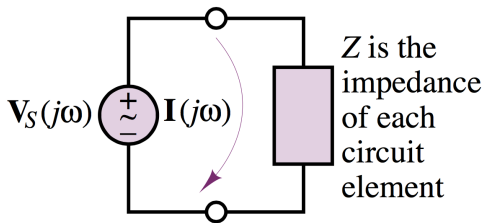






## Impedance

A complex resistance or *frequency-dependent resistance*. That is, as resistors whose resistance is a function of the frequency of the sinusoidal excitation.



AC circuits in  
phasor/impedance form

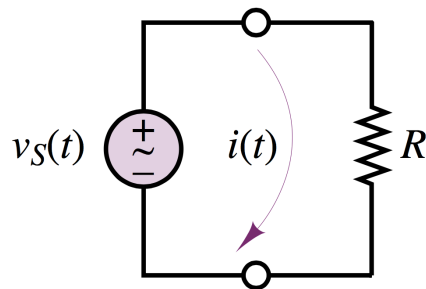
# Resistor Impedance



Assume source voltage  $v = A \cos(\omega t)$ , then

▶  $\mathbf{V}(j\omega) = A\angle 0$

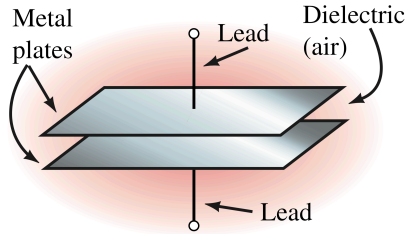
▶  $\mathbf{I}(j\omega) = \frac{A}{R}\angle 0$



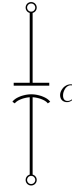
## Impedance of A Resistor

$$Z_R(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)} = R\angle 0 = R$$

# Capacitor ABC



(a) Basic construction



(b) Symbol

## Capacitance $C$

A measure of how much charge a capacitor can hold.

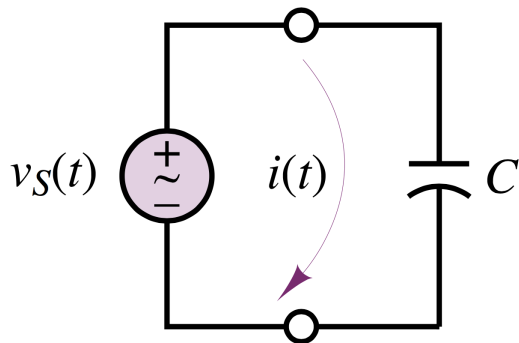
▶ Amount of charge  $Q = C \cdot V$

▶ **current** is the rate of movement of charge:  $I = \frac{dQ}{dt} = C \cdot \frac{dV}{dt}$

# Capacitor Impedance



$$\mathbf{V}(j\omega) = A\angle 0$$
$$\mathbf{I}(j\omega) = \omega CA\angle \pi/2$$



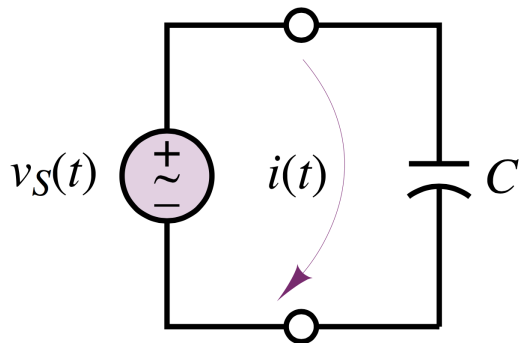
## Impedance of A Capacitor

$$Z_C(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)}$$

# Capacitor Impedance



$$\mathbf{V}(j\omega) = A\angle 0$$
$$\mathbf{I}(j\omega) = \omega CA\angle \pi/2$$

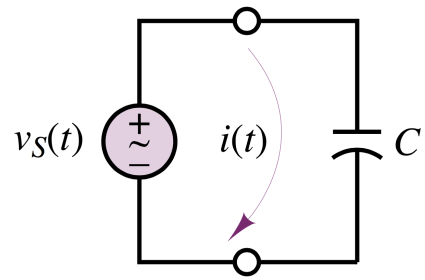


## Impedance of A Capacitor

$$Z_C(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)}$$



$$Z_C(j\omega) = \frac{1}{j\omega C}$$



### Capacitor Rule 1

Low Frequency  $\Rightarrow$  Open circuit

### Capacitor Rule 2

High Frequency  $\Rightarrow$  Short circuit

# Overview

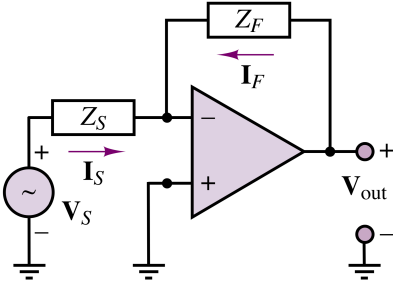


Preliminaries

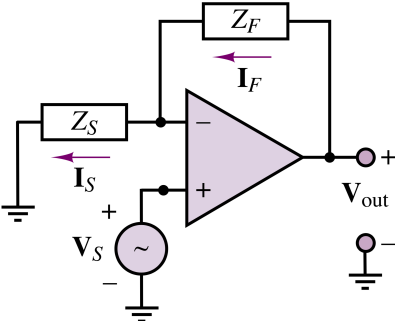
Integrator & Differentiator

Filters

# Frequency Response of An Op-Amp



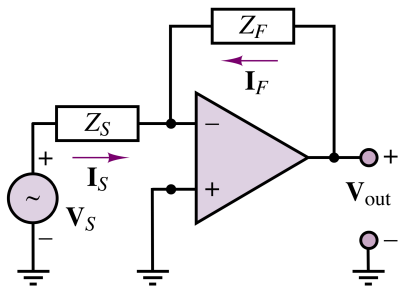
Inverting



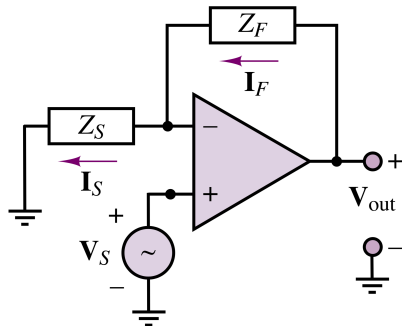
Noninverting



# Frequency Response of An Op-Amp



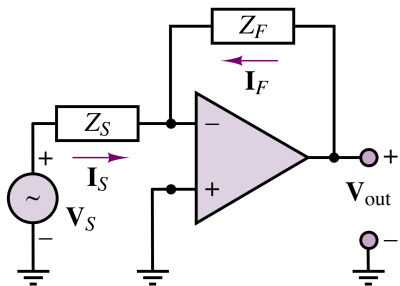
Inverting



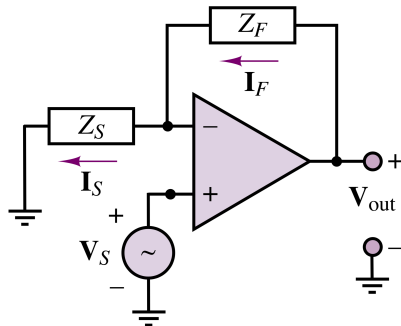
Noninverting

- ▶ Inverting amplifier:  $\frac{V_{out}(j\omega)}{V_S} = -\frac{Z_F}{Z_S}$

# Frequency Response of An Op-Amp



Inverting

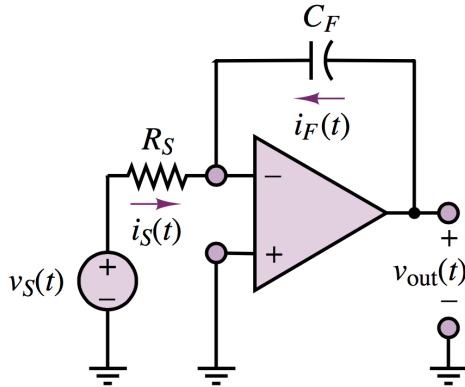


Noninverting

▶ Inverting amplifier: 
$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

▶ Non-Inverting amplifier: 
$$\frac{V_{out}}{V_S}(j\omega) = 1 + \frac{Z_F}{Z_S}$$

# Integrator

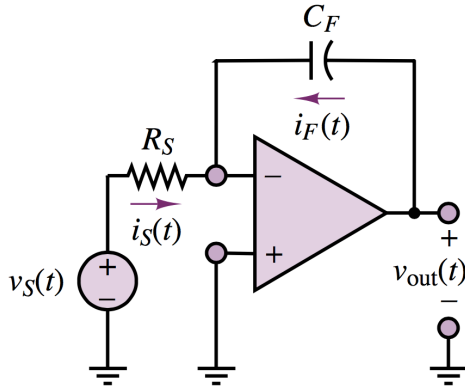


$$i_S(t) = -i_F(t)$$

$$i_S(t) = \frac{v_S(t)}{R_S}$$

$$i_F(t) = C_F \cdot \frac{dv_{out}(t)}{dt}$$

# Integrator



$$i_S(t) = -i_F(t)$$

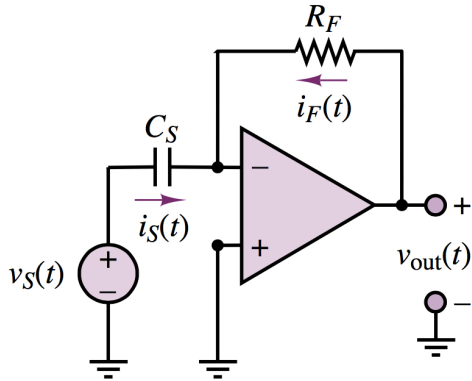
$$i_S(t) = \frac{v_S(t)}{R_S}$$

$$i_F(t) = C_F \cdot \frac{dv_{out}(t)}{dt}$$

Therefore:

$$v_{out}(t) = -\frac{1}{R_S C_F} \int_{-\infty}^t v_S(t') dt'$$

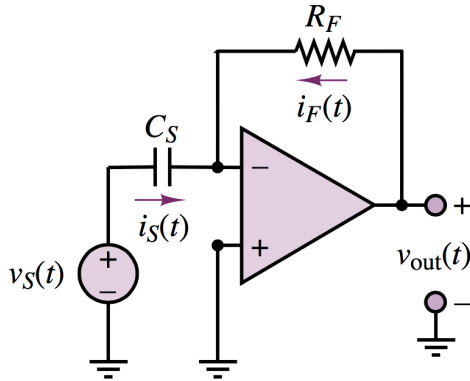
# Differentiator



$$i_S(t) = C_S \cdot \frac{dv_S(t)}{dt}$$

$$i_F(t) = \frac{v_{out}(t)}{R_F}$$

# Differentiator



$$i_S(t) = C_S \cdot \frac{dv_S(t)}{dt}$$

$$i_F(t) = \frac{v_{out}(t)}{R_F}$$

Therefore:

$$v_{out}(t) = -R_F C_S \cdot \frac{dv_S(t)}{dt}$$

# Overview

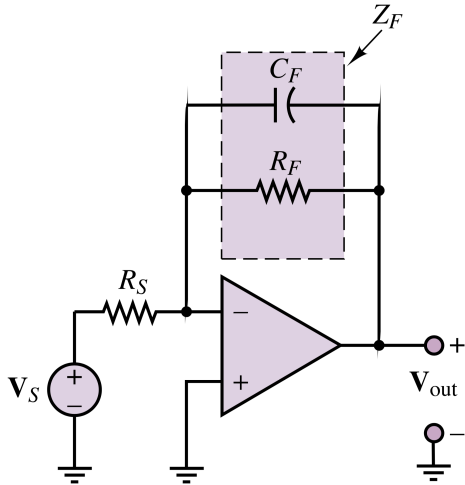


Preliminaries

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# Low-Pass Filter



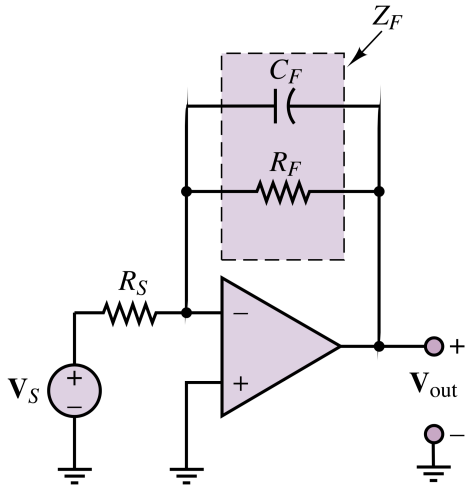
$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F}$$

$$Z_S = R_S$$



# Low-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F}$$

$$Z_S = R_S$$



$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F/R_S}{1 + j\omega C_F R_F}$$



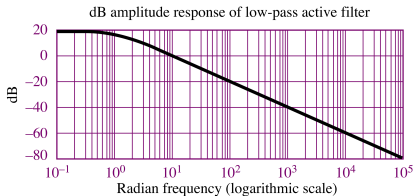
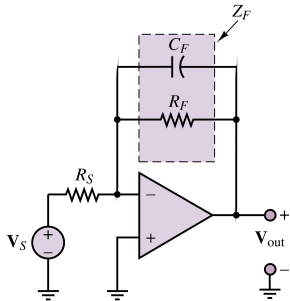
Given:

$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F/R_S}{1 + j\omega C_F R_F}$$
$$w_c = \frac{1}{R_F C_F}$$

Prove:

$$|A| = \frac{R_F}{R_S} \cdot \frac{1}{\sqrt{1 + \omega^2/w_c^2}}$$

# Low-Pass Filter

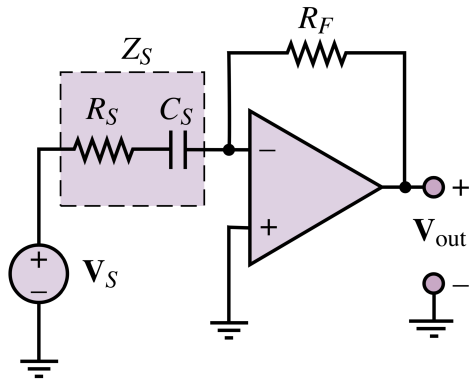


$$|A| = \frac{R_F}{R_S} \cdot \frac{1}{\sqrt{1 + \omega^2/w_c^2}}$$

- ▶  $w_c = \frac{1}{R_F C_F}$
- ▶ **3-dB** frequency
- ▶ or **cutoff** frequency

BTW,  $\lim_{\omega \rightarrow 0} |A| = \frac{R_F}{R_S}$ ,  $\lim_{\omega \rightarrow \infty} |A| = 0$

# High-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

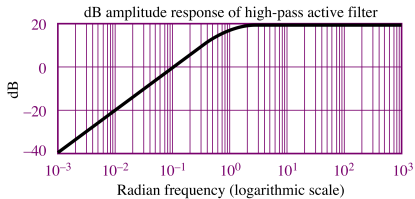
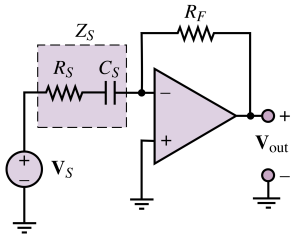
$$Z_S = R_S + \frac{1}{j\omega C_S}$$

$$Z_F = R_F$$



$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega C_S R_S}$$

# High-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega C_S R_S}$$

$$\lim_{\omega \rightarrow 0} |A| = 0$$

$$\lim_{\omega \rightarrow \infty} |A| = \frac{R_F}{R_C}$$

High freq. cutoff unintentionally created by Op-amp

# Band-Pass Filter

