CENG 5030
Energy Efficient Computing

L02: CNN Training

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Overview

CNN Intialization

Gradient Descent Methods

Learning rate annealing

Normalization

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Gradient Descent Methods

Learning rate annealing

Normalization

CNN Initialization

- Constant initialization or Random initialization
- Orthogonal initialization
- Xaiver initialization
- Kaiming initialization

Initialization methods contain/adapt materials developed by

- Kaiming He et al. (Dec. 2015). "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification". In: *Proceedings of the IEEE International Conference on Computer Vision (ICCV)*
- Andrew M. Saxe, James L. McClelland, and Surya Ganguli (2013). *Exact solutions to the nonlinear dynamics of learning in deep linear neural networks*. eprint: arXiv:1312.6120
- Xavier Glorot and Yoshua Bengio (13–15 May 2010). "Understanding the difficulty of training deep feedforward neural networks". In: ed. by Yee Whye Teh and Mike Titterington. Vol. 9. Proceedings of Machine Learning Research. Chia Laguna Resort, Sardinia, Italy: JMLR Workshop and Conference Proceedings, pp. 249–256. URL: http://proceedings.mlr.press/v9/glorot10a.html

Constant initialization or Random initialization

- Constant Initialization

Constant Initialization

All weights are initialized by a constant
This will reduce the learning efficiency and generalization ability of a neural network

- Random initialization

Uniform distribution intialization[1]

$$W \sim U[a, b]$$

Gaussian initialization

$$W \sim Normal(\mu, \sigma^2)$$

[1] https://www.ucd.ie/msc/t4media/Uniform%20Distribution.pdf

Xaiver Initialization

Definition

z^i is the input vector of layer i, and s^i is the feature vector of the activation function at layer i, we have $s^i = z^i \cdot W^i + b^i$ and $z^{i+1} = f(s^i)$ where f is the symmetric activation function with unit derivative at 0 (i.e., $f'(0) = 1$)

Insight

For the stable mapping from the input domain to the target domain, we want to make the standard derivation of the input domain and the target domain are as equal as possible (i.e., keep the information flowing better in the neural networks)

Forward propagation

$$Var(z^i) = Var(x) \cdot \prod_{i'=0}^{i-1} n_{i'} Var(W^{i'})$$

Backward propagation

$$Var(\frac{\partial Cost}{\partial s^i}) = Var(\frac{\partial Cost}{\partial s^d}) \cdot \prod_{i'=i}^{d} n_{i'+1} Var(W^{i'})$$

Xaiver Initialization

Forward propagation

$$\forall(i), n_i Var(W^i) = 1$$

Backward propagation

$$\forall(i), n_{i+1} Var(W^i) = 1$$

As a compromise between two constraints

$$\forall(i), Var(W^i) = \frac{2}{n_i + n_{i+1}}$$

Assume we use a Uniform distribution

$$W \sim U[-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}]$$

Xaiver inititialization

$$W \sim U[-\frac{\sqrt{6}}{\sqrt{n_i + n_{i+1}}}, \frac{\sqrt{6}}{\sqrt{n_i + n_{i+1}}}]$$

Kaiming Initialization

Insight

W has a symmetric distribution around zero and $E[b] = 0$

Follow the same rule with Xaiver initiailization

We have

$$Var(z^i) = \frac{1}{2} Var(x^i)$$

Kaiming initialization

$$W \sim U[-\sqrt{\frac{6}{n_i}}, \sqrt{\frac{6}{n_{i+1}}}]$$

Orthogonal Initialization

Insight

Solve the problem of gradient varnishing and exloding in Recurrent Neural Networks

Assume that we are taking a RNN as a form below

A form of a RNN

$$y_t = W^t \cdot y_0$$

We apply orthogonal decomposition to W

We have

$$y_t = Q\Lambda^t Q \cdot y_0$$

Since an orthogonal matrix has the eigenvalue of one, so the gradients would not varnish or explode during the training phase

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Basic gradient descent methods

- Batch gradient descent
- Stochastic gradient descent
- Mini-batch gradient descent

Reference

- Sebastian Ruder (2016). *An overview of gradient descent optimization algorithms*. eprint: arXiv:1609.04747

Batch gradient descent

The gradient of the cost function w.r.t. the parameters θ

$$\theta = \theta - \eta \cdot \bigtriangledown_\theta J(\theta)$$

Features

- The algorithm performs slowly
- The algorithm cannot update the model online (i.e., with new examples on-the-fly)
- The datasets cannot fit in memory possibly

Tips

Batch gradient descent is guaranteed to converge to the global minimum for convex error surfaces and to a local minimum for non-convex surfaces

Stochastic gradient descent

The gradient of the cost function w.r.t. the parameters θ

$$\theta = \theta - \eta \cdot \bigtriangledown_\theta J(\theta; x^i, y^i)$$

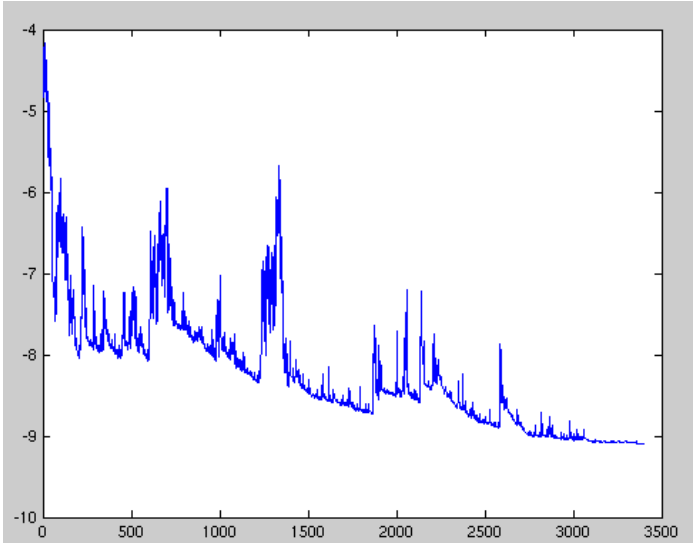
Features

- The algorithm usually much faster
- The algorithm can be used to learn online
- SGD cause the objective function to fluctuate heavily since it performs frequent updates with a high variance

Tips

The fluctuation can help to jump to new and potentially better local minima

Stochastic graident descent[2]



[2]By Joe pharos at the English language Wikipedia, CC BY-SA 3.0

Mini-batch gradient descent

The gradient of the cost function w.r.t. the parameters θ

$$\theta = \theta - \eta \cdot \bigtriangledown_{\theta} J(\theta; x^{(i:i+n)}, y^{(i:i+n)})$$

Features

- Reduce the variance of the parameter updates, which can lead to more stable convergence
- Make use of highly optmized matrix optmizations libraries to accelerate the gradient (i.e., very efficient)

Tips

Most of training methods utilize mini-batch gradient descent method

Challenges

Challenges

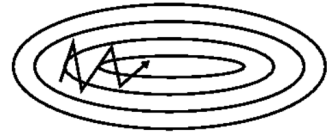
- It is difficult to decide a proper learning rate
- It is difficult to decide a proper learning rate schedule
- It is not reasonable to update all parameters using the same learning rate
- It is difficult to jump out of local minima and also saddle points

Gradient descent optimization methods

Gradient descent optmization methods contain/adapt materials developed by

- Ning Qian (1999). “On the momentum term in gradient descent learning algorithms”. In: *Neural networks* 12.1, pp. 145–151
- Yurii E Nesterov (1983). “A method for solving the convex programming problem with convergence rate O (1/kˆ 2)”. In: *Dokl. akad. nauk Sssr*. Vol. 269, pp. 543–547
- Matthew D Zeiler (2012). “Adadelta: an adaptive learning rate method”. In: *arXiv preprint arXiv:1212.5701*
- John Duchi, Elad Hazan, and Yoram Singer (2011). “Adaptive subgradient methods for online learning and stochastic optimization.”. In: *Journal of machine learning research* 12.7
- Diederik P Kingma and Jimmy Ba (2014). “Adam: A method for stochastic optimization”. In: *arXiv preprint arXiv:1412.6980*

Momentum



3

$$v_t = \gamma v_{t-1} + \eta \bigtriangledown_\theta J(\theta)$$
$$\theta = \theta - v_t$$

Features

- The momentum increases for dimensions whose gradients point in the same directions
- The momentum reduces updates for dimensions whose gradients change directions

Tips

The momentum term γ is usually set to 0.9 or a similar value

[3] Genevieve B. Orr, Willamette University cs449.

Nesterov

$$v_t = \gamma v_{t-1} + \eta \bigtriangledown_\theta J(\theta - \gamma v_{t-1})$$
$$\theta = \theta - v_t$$

Features

- NAG uses predicted update to prevent us from going too fast compared with Momentum
- It increases the performance of RNNs on a number of tasks

Adagrad

$$\theta_{t+1,i} = \theta_{t,i} - \frac{\eta}{\sqrt{G_{t,ii} + \epsilon}} \cdot g_{t,i}$$
$$g_{t,i} = \bigtriangledown_{\theta_t} J(\theta_{t,i})$$

where $G_t \in \mathbb{R}^{d \times d}$ is a diagonal matrix where each diagonal elment $G_{t,ii}$ is the sum of squares of the gradients w.r.t. θ_i up to time step t, ϵ is a smoothing term which avoids division by zero (e.g., $1e-8$)

Features

- The learning rate η for different parameters can be varied based on the past gradients
- The accumulated sum of the denominator keeps growing during training, which may cause gradient varnishing

Tips

The initial learing rate η of most applications uses 0.01 as a default value

Adadelta

$$\theta_{t+1,i} = \theta_{t,i} + \Delta\theta_{t,i}$$
$$\Delta\theta_{t,i} = -\frac{RMS[\Delta\theta]_{t-1,i}}{RMS[g]_{t,i}} g_{t,i}$$

where

$$RMS[\Delta\theta]_{t,i} = \sqrt{E[\Delta\theta^2]_{t,i} + \epsilon}$$
$$E[\Delta\theta^2]_{t,i} = \gamma E[\Delta\theta^2]_{t-1,i} + (1-\gamma)\Delta\theta^2_{t,i}$$

Tips

With Adadelta, we do not need to set a default learning rate, since it has benn eliminated from the update rule

RMSprop

$$E[g^2]_t = \gamma E[g^2]_{t-1} + (1-\gamma) g_t^2$$
$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{E[g^2]_t + \epsilon}} g_t$$

Tips

Hinton suggests γ to be set to 0.9 and a good default value for the learning rate η is 0.001 [4]

[4] https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

Adam

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v} + \epsilon}} \hat{m}_t$$

where

$$\hat{m}_t = \frac{m_t}{1 - \beta_1^t}$$
$$\hat{v}_t = \frac{v_t}{1 - \beta_2^t}$$
$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$
$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

Tips

The authors propse the default value of β_1 to be 0.9, 0.999 for β_2 and 10^{-8} for ϵ

Which optimizer to choose?

- Choose one of the adaptive learning-rate optimization methods usually obtain better performance and you will not need to tune the learning rate with the default value
- Adam might be the best overall choice
- Vanilla SGD is robust to different initialization methods and learning rate annealing schedulers

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Different annealing methods

- Step
- Cosine
- Cyclical

Learning rate annealing methods contain/adapt materials developed by

- Leslie N Smith (2017). “Cyclical learning rates for training neural networks”. In: *2017 IEEE Winter Conference on Applications of Computer Vision (WACV)*. IEEE, pp. 464–472
- Ilya Loshchilov and Frank Hutter (2016). “Sgdr: Stochastic gradient descent with warm restarts”. In: *arXiv preprint arXiv:1608.03983*

Step learning rate scheduler

Step learning rate scheduler

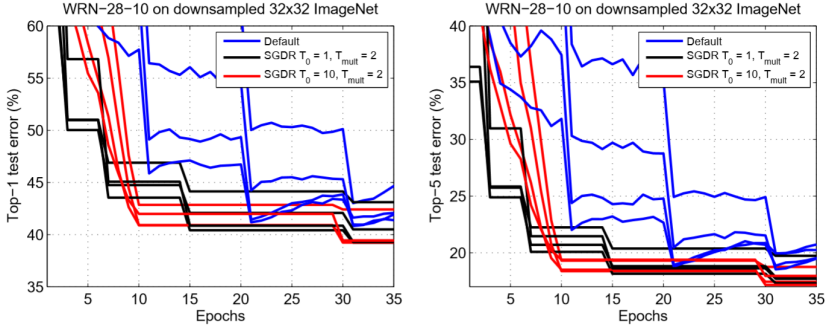
Decay the learning rate by a pre-defined step size exponentially characterized by γ

Cosine learning rate scheduler

Cosine annealing method / SGDR (i.e., Stochastic Gradient Descent with warm Restarts)

$$\eta_t = \eta^t_{min} + \frac{1}{2}(\eta^i_{max} - \eta^i_{min})(1 + \cos(\frac{T_{cur}}{T_i}\pi))$$

Where T_i represents the total epoch numbers

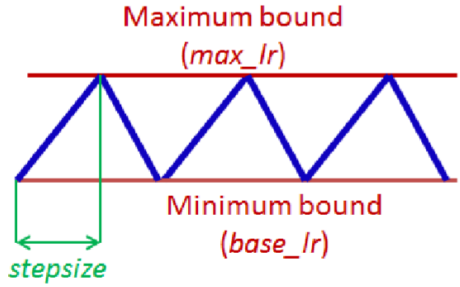


Top-1 and Top-5 test errors obtained by SGD with momentum (i.e., Default) and SGDR on WRN-28-10 trained on a version of ImageNet

Cyclical learning rate scheduler

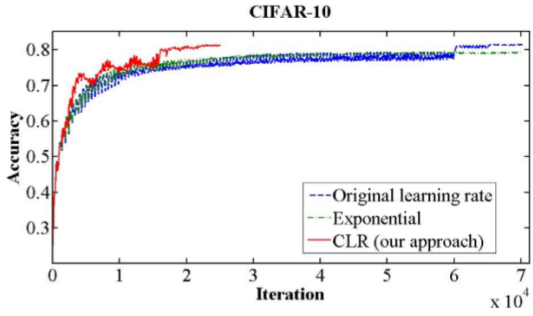
Insight

The essence of this learning rate policy comes from the observation that increasing the learning rate might have a short term negtive effect and yet achieve a longer term beneficial effect



Triangular learning rate policy

Cyclical learning rate scheduler

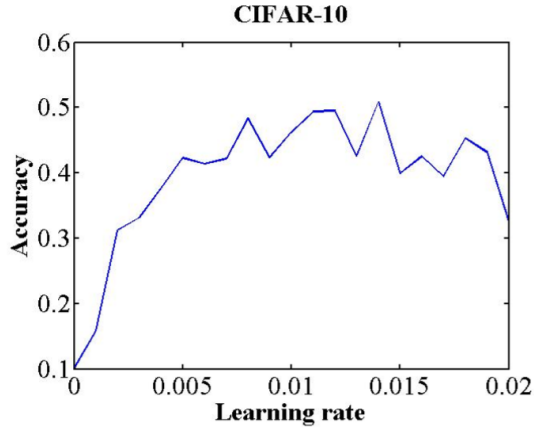


Classification accuracy while training CIFAR-10

Tips

- Experiments show that it often is good to set **stepsize** equal to 2-10 times the number of iterations in an epoch (e.g., $stepsize = 8 \cdot epoch$)
- Use **LR range test** to estimate reasonable minimum and maximum boundary values

Cyclical learning rate scheduler



Classification accuracy as a function of increasing learning rate for 8 epochs

The figure shows that the model starts to converge from the beginning, so it is reasonable to set the baseline learning rate to 0.001. Furthermore, when the learning rate is above 0.006, the accuracy rise gets rough and eventually begins to drop so it is reasonable to set the maximum learning rate to 0.006

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Normalization

Reduce **Internal Covariate Shift**, accelerate the training and improve the performance of a neural network

- Batch normalization
- Layer normalization
- Instance normalization
- Weight normalization

Normalization methods contain/adapt materials developed by

- Sergey Ioffe and Christian Szegedy (2015). "Batch normalization: Accelerating deep network training by reducing internal covariate shift". In: *arXiv preprint arXiv:1502.03167*
- Jimmy Lei Ba, Jamie Ryan Kiros, and Geoffrey E Hinton (2016). "Layer normalization". In: *arXiv preprint arXiv:1607.06450*
- Dmitry Ulyanov, Andrea Vedaldi, and Victor Lempitsky (2016). "Instance normalization: The missing ingredient for fast stylization". In: *arXiv preprint arXiv:1607.08022*
- Tim Salimans and Durk P Kingma (2016). "Weight normalization: A simple reparameterization to accelerate training of deep neural networks". In: *Advances in neural information processing systems*, pp. 901–909

Batch normalization

Internal Covariate Shift

Internal Covariate Shift is as the change in the distribution of network activations due to the change in network parameters during training

Algorithm 1 Batch Normalization Transform

Input: Values of x over a mini-batch: $\mathcal{B} = x_{1,2,\ldots,m}$;

Parameters to be learned: γ, β

Output: $y_i = BN_{\gamma,\beta}(x_i)$

// mini-batch mean

$\mu_{\mathcal{B}} = \frac{1}{m}\sum_{i=1}^{m} x_i$

// mini-batch variance

$\sigma^2_{\mathcal{B}} = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2$

// normalize

$\hat{x_i} = \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma^2_{\mathcal{B}} + \epsilon}}$

// scale and shift

$y_i = \gamma\hat{x}_i + \beta \equiv BN_{\gamma,\beta}(x_i)$

Batch normalization

- Back propagation with Batch Normalization
- Use chain rules to optmize γ and β:

$$\frac{\partial \ell}{\partial \hat{x}_i} = \frac{\partial \ell}{\partial y_i} \cdot \gamma$$

$$\frac{\partial \ell}{\partial \sigma_{\mathcal{B}}^2} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial \hat{x}_i} \cdot (x_i - \mu_{\mathcal{B}}) \cdot \frac{-1}{2} (\sigma_{\mathcal{B}}^2 + \epsilon)^{\frac{-3}{2}}$$

$$\frac{\partial \ell}{\partial \mu_{\mathcal{B}}} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial \hat{x}_i} \cdot \frac{-1}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$$

$$\frac{\partial \ell}{\partial \hat{x}_i} \cdot \frac{-1}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} + \frac{\partial \ell}{\partial \sigma_{\mathcal{B}}^2} \cdot \frac{2(x_i - \mu_{\mathcal{B}})}{m} + \frac{\partial \ell}{\partial \mu_{\mathcal{B}}} \cdot \frac{1}{m}$$

$$\frac{\partial \ell}{\partial \gamma} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial y_i} \cdot \hat{x}_i$$

$$\frac{\partial \ell}{\partial \beta} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial_i}$$

Batch normalization

Algorithm 2 Training a Batch Normalization Network

Input: Network N with trainable parameters Θ;

1: subset of activations $x^k{}_{k=1}^K$

Output: Batch-normalized network for inference, N_{BN}^{inf}

2: $N_{BN}^{tr} \leftarrow N$ // Training BN network

3: **for** $k = 1$ to K **do**

4: Add transformation $y^{(k)} = BN_{\gamma^k,\beta^{(k)}}(x^{(k)})$ to N_{BN}^{tr} (i.e., Algorithm of Batch Normalization Transform)

5: Modify each layer in N_{BN}^{tr} with input $x^{(k)}$ to take $y^{(k)}$ instead

6: **end for**

7: Train N_{BN}^{tr} to optimize the parameters $\Theta \bigcup \gamma^k, \beta^k{}_{k=1}^K$

8: $N_{BN}^{inf} \leftarrow N_{BN}^{tr}$ // Inference BN network with frozen parameters

9: **for** $k = 1$ to K **do**

10: // For clarity, $x \equiv x^{(k)}, \gamma \equiv \gamma^{(k)}, \mu_{\mathcal{B}} \equiv \mu_{\mathcal{B}}^{(k)}$

11: Process multiple training mini-batches $\mathcal{B}$, each of size m, and average over them:

12: $E[x] \leftarrow E_{\mathcal{B}}[\mu_{\mathcal{B}}]$

13: $VAR[x] \leftarrow \frac{m}{m-1} E_{\mathcal{B}}[\sigma_{\mathcal{B}}^2]$

14: In N_{BN}^{inf}, replace the transform $y = BN_{\gamma,\beta}(x)$ with $y = \frac{\gamma}{\sqrt{VAR[x]+\epsilon}} \cdot x + (\beta - \frac{\gamma E[x]}{\sqrt{VAR[x]+\epsilon}})$

15: **end for**

Weight normalization

$$w = \frac{g}{\|v\|} v$$

where the gradients of g and v:

$$\nabla_g L = \frac{\nabla_w L \cdot v}{\|v\|}$$

$$\nabla_v L = \frac{g}{\|v\|} \nabla_w L - \frac{g \nabla_w L}{\|v\|^2} v$$

The relationship between Batch normalization

The pre-activation t for each mini-batch in the Batch normalization

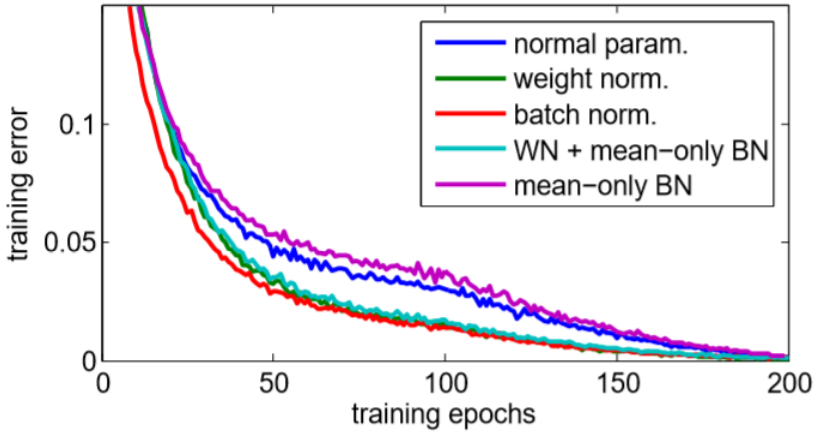
$$t' = \frac{t - \mu[t]}{\sigma[t]}$$

and

$$t = v \cdot x$$

For the special case of a single layer with the input features x that has been whitened (i.e., $\mu[x] = 0$ and $\sigma[x] = 1$) Weight normalization is given by $\mu[t] = 0$ and $\sigma[t] = \|v\|$

Weight normalization



Training error for CIFAR-10 using different network parameterizations

Layer normalization

$$a^t = W_{hh}h^{t-1} + W_{xh}x^t$$
$$h^t = f[\frac{g}{\sigma^t} \bigodot (a^t - \mu^t) + b]$$
$$\mu^t = \frac{1}{H}\sum_{i=1}^{H} a_i^t$$
$$\sigma^t = \sqrt{\frac{1}{H}\sum_{i=1}^{H}(a_i^t - \mu^t)^2}$$

where x^t is the current input, h^{t-1} is the previous vector of hidden states. W_{hh} is the recurrent hidden to hidden weights, W_{xh} is the bottom up input to hidden weights, $\bigodot$ is the element-wise multiplication between two vectors, b and g are defined as the bias and gain parameters of the same dimension as h^t

Features

- Layer normalization does not calculate the mean and standard derivation based on the mini-batch, which can be utilized in RNN rather than Batch normalization
- All the hidden units share the same normalization terms μ and σ
- Layer normalization does not impose any constraint on the size of a mini-batch and it can be used in the pure online regime with batch size 1

Instance normalization

Let $x \in \mathbb{R}^{T \times C \times W \times H}$ and x_{tijk} denote its $tijk$-th element where j and k span spatial dimensions, i is the feature channel (color channel if the input is an RGB image), and t is the index of the image in the batch. The instance normalization is formulated as:

$$y_{tijk} = \frac{x_{tijk} - \mu_{ti}}{\sqrt{\sigma_{ti}^2 + \epsilon}}$$

$$\mu_{ti} = \frac{1}{HW} \sum_{l=1}^{W} \sum m = 1^{H} x_{tilm}$$

$$\sigma_{ti}^2 = \frac{1}{HW} \sum_{l=1}^{W} \sum_{m=1}^{H} (x_{tilm} - \mu_{ti})^2$$

Features

- Instance normalization is used in the style transfer task (i.e., task-specific technique)
- It is based on Batch normalization

Instance normalization

Stylization examples using instance normalization. First row: style images, second row: original images and its stylized versions