

CENG4480

Lecture 03: Operational Amplifier – 2

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Overview

Preliminaries

Integrator & Differentiator

Filters



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Biasing

Biasing in electronics

The method of establishing predetermined voltages or currents at various points of an electronic circuit for the purpose of establishing proper operating conditions in electronic components

<https://en.wikipedia.org/wiki/Biasing>



Eulers Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

- ▶ real component
- ▶ imaginary component
- ▶ magnitude

$$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$



Prove:

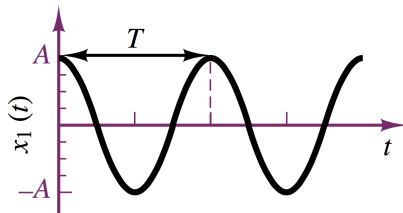
$$\left| \frac{1}{1 + ja} \right| = \frac{1}{\sqrt{1 + a^2}}$$



Sinusoidal Signal

$$x(t) = A\cos(\omega t + \phi)$$

- ▶ Periodic signals
- ▶ A : amplitude
- ▶ ω : radian frequency
- ▶ ϕ : phase

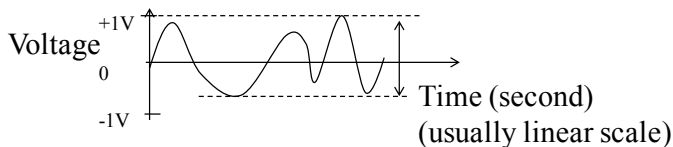


Time Domain

- ▶ Voltage gain against **time**

For sinusoidal signal:

$$v(t) = A\cos(\omega t + \phi)$$

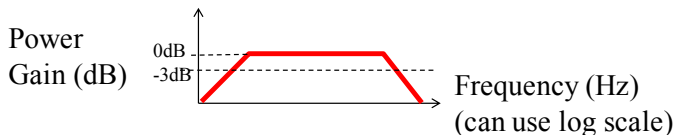


Frequency Domain

- ▶ Voltage gain against frequency

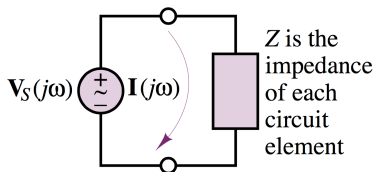
For sinusoidal signal:

$$\mathbf{V}(j\omega) = Ae^{j\phi} = A\angle\phi$$



Impedance

A complex resistance or *frequency-dependent resistance*. That is, as resistors whose resistance is a function of the frequency of the sinusoidal excitation.

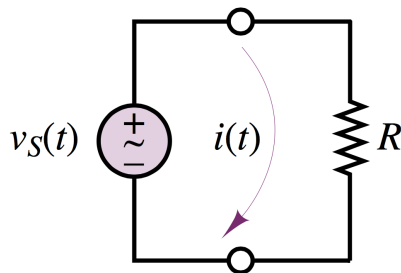


AC circuits in
phasor/impedance form



Resistor Impedance

- ▶ $\mathbf{V}(j\omega) = A\angle 0$
- ▶ $\mathbf{I}(j\omega) = \frac{A}{R}\angle 0$

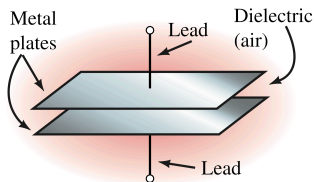


Impedance of A Resistor

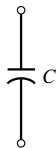
$$Z_R(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)} = R$$



Capacitor ABC



(a) Basic construction



(b) Symbol

Capacitance C

A measure of how much charge a capacitor can hold.

- ▶ Amount of charge $Q = C \cdot V$

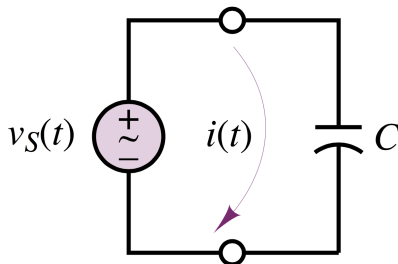
- ▶ **current** is the rate of movement of charge: $I = \frac{dQ}{dt} = C \cdot \frac{dV}{dt}$



Capacitor Impedance

$$\mathbf{V}(j\omega) = A\angle 0$$

$$\mathbf{I}(j\omega) = \omega CA\angle \pi/2$$

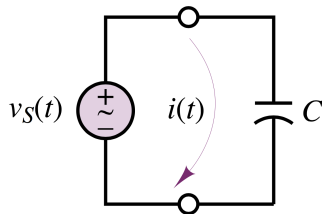


Impedance of A Capacitor

$$Z_C(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)} =$$



$$Z_C(j\omega) = \frac{1}{j\omega C}$$



Capacitor Rule 1

Low Frequency \Rightarrow Open circuit

Capacitor Rule 2

High Frequency \Rightarrow Short circuit



Overview

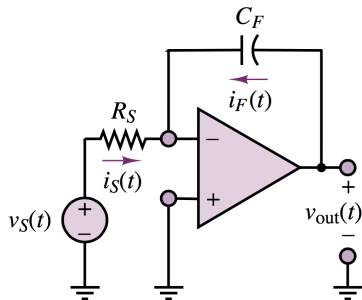
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Integrator



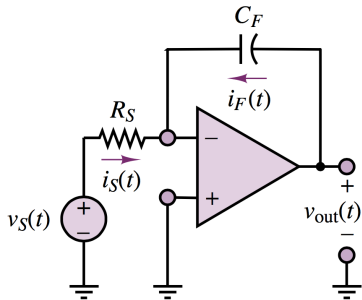
$$i_S(t) = -i_F(t)$$

$$i_S(t) = \frac{v_S(t)}{R_S}$$

$$i_F(t) = C_F \cdot \frac{dv_{out}(t)}{dt}$$



Integrator



$$i_S(t) = -i_F(t)$$

$$i_S(t) = \frac{v_S(t)}{R_S}$$

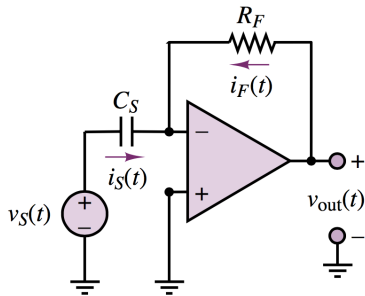
$$i_F(t) = C_F \cdot \frac{dv_{out}(t)}{dt}$$

Therefore:

$$v_{out}(t) = -\frac{1}{R_S C_F} \int_{-\infty}^t v_S(t') dt'$$



Differentiator

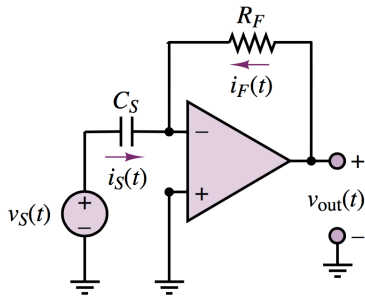


$$i_S(t) = C_S \cdot \frac{dv_S(t)}{dt}$$

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Differentiator



$$i_S(t) = C_S \cdot \frac{dv_S(t)}{dt}$$

$$i_F(t) = \frac{v_{out}(t)}{R_F}$$

Therefore:

$$v_{out}(t) = -R_F C_S \cdot \frac{dv_S(t)}{dt}$$



Overview

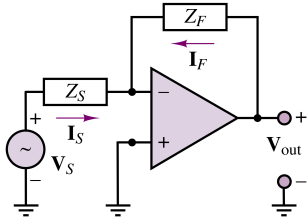
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Integrator & Differentiator

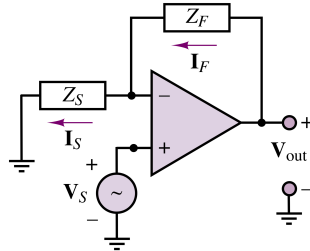
Filters



Frequency Response of An Op-Amp



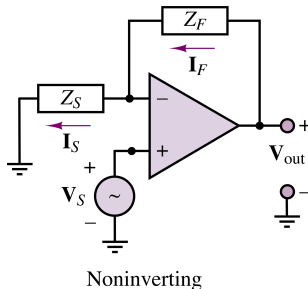
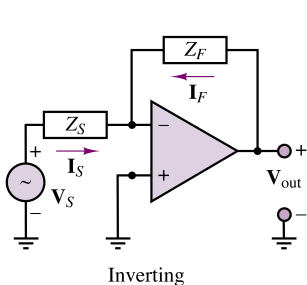
Inverting



Noninverting



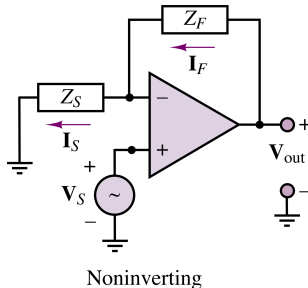
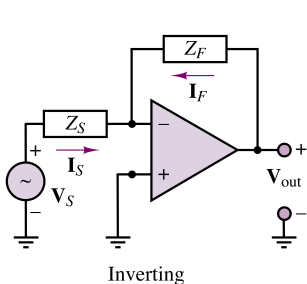
Frequency Response of An Op-Amp



- ▶ Inverting amplifier: $\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$



Frequency Response of An Op-Amp

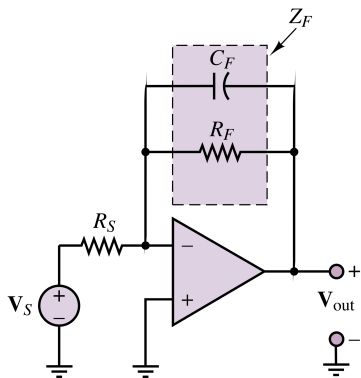


▶ Inverting amplifier: $\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$

▶ Non-Inverting amplifier: $\frac{V_{out}}{V_S}(j\omega) = 1 + \frac{Z_F}{Z_S}$



Low-Pass Filter



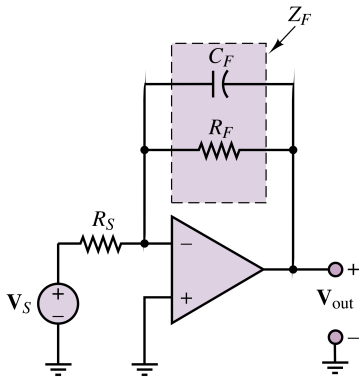
$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F}$$

$$Z_S = R_S$$



Low-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F}$$

$$Z_S = R_S$$

⇒

$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F/R_S}{1 + j\omega C_F R_F}$$



Given:

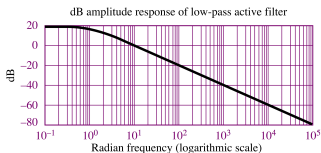
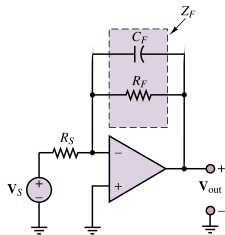
$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F/R_S}{1 + j\omega C_F R_F}$$
$$w_c = \frac{1}{R_F C_F}$$

Prove:

$$|A| = \frac{R_F}{R_S} \cdot \frac{1}{\sqrt{1 + w^2/w_c^2}}$$



Low-Pass Filter



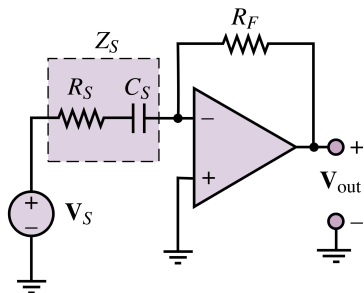
$$\text{BTW, } \lim_{\omega \rightarrow 0} |A| = \frac{R_F}{R_S}, \lim_{\omega \rightarrow \infty} |A| = 0$$

$$|A| = \frac{R_F}{R_S} \cdot \frac{1}{\sqrt{1 + \omega^2 / \omega_c^2}}$$

- ▶ $\omega_c = \frac{1}{R_F C_F}$
- ▶ **3-dB** frequency
- ▶ or **cutoff** frequency



High-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_S = R_S + \frac{1}{j\omega C_S}$$

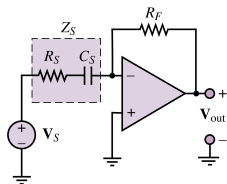
$$Z_F = R_F$$

⇒

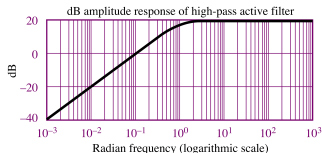
$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega C_S R_S}$$



High-Pass Filter



$$A(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega C_S R_S}$$



$$\lim_{\omega \rightarrow 0} |A| = 0$$

$$\lim_{\omega \rightarrow \infty} |A| = \frac{R_F}{R_C}$$

High freq. cutoff unintentionally created by Op-amp



Band-Pass Filter

