# CENG4480 Homework 1

- **Small-Signal Gain:** For given amp circuits, small changes of input $\Delta V_{in}$ will cause output change of $\Delta V_{out}$. Small-signal gain is defined by $\frac{\Delta V_{out}}{\Delta V_{in}}$.

## Solutions

**Q1** Given a non-inverting amplifier as shown in Fig. 1, calculate the exact finite gain. Assume $A_0 = 1000$, determine the gain difference if the circuit is expected to have an ideal gain of 5 under $A_0 = \infty$.

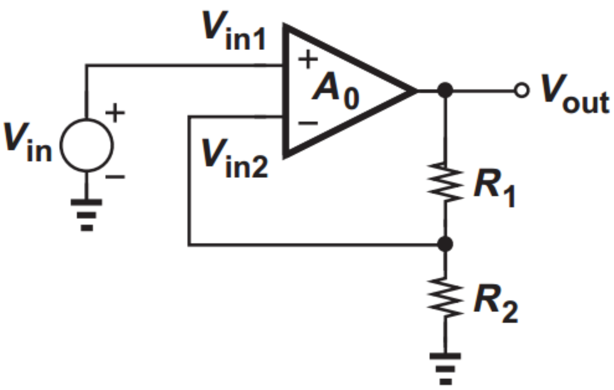


Figure 1: Non-inverting Amplifier

**A1** From the proterties of Op Amplifier,

$$V_{out} = A_0(V_{in1} - V_{in2}) \tag{1}$$

Given that,

$$V_{in2} = \frac{R_2}{R_1 + R_2} V_{out} \tag{2}$$

Substituting into (1) we have,

$$g_{real} = \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \frac{R_2}{R_1+R_2} A_0} \tag{3}$$

Besides,

$$g_{ideal} = (1 + \frac{R_1}{R_2}) \tag{4}$$

Substituting data into Eqs. (3) and (4),

$$g_{real} = 4.975, g_{ideal} = 5 \tag{5}$$

Thus, real circuit gain has a $0.5\%$ difference from ideal gain.

**Q2** An op amp exhibits the following nonlinear characteristic:

$$V_{out} = \alpha \tanh[\beta(V_{in1} - V_{in2})]. \tag{6}$$

Determine the small-signal gain of the op amp in the case $V_{in1} \approx V_{in2}$.

**A2** Taylor expansion of tanh has the following form:

$$\tanh(Z) = Z - \frac{1}{3}Z^3 + \frac{2}{15}Z^5 - ... \tag{7}$$

When $V_{in1} \approx V_{in2}$,

$$V_{out} \approx \alpha\beta(V_{in1} - V_{in2}) \tag{8}$$

Thus, small-signal gain is,

$$\frac{dV_{out}}{d(V_{in1} - V_{in2})} = \alpha\beta \tag{9}$$

**Q3** Assuming $A_0 = \infty$, compute the closed-loop gain of the inverting op amp shown in Fig. 2. Verify that the result reduces to ideal version when $R_1 \to 0$.

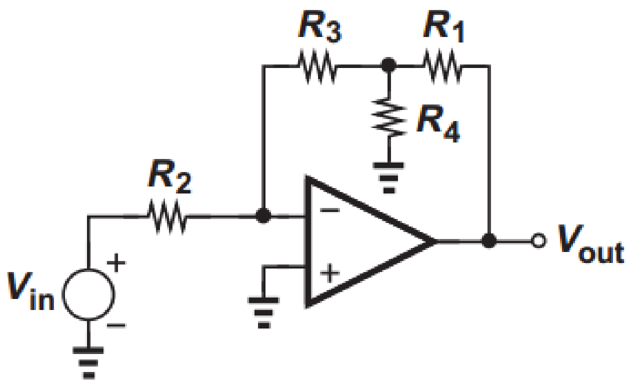


Figure 2: Inverting Op Amp

**A3** Given infinite open-loop gain,

$$V_- = V_+ = 0 \tag{10}$$

Let $V_x$ be the voltage of the point where $R_1$, $R_3$ and $R_4$ crossed, then,

$$\frac{V_{in}}{R_2} = \frac{V_x}{R_3} \tag{11}$$

In addition,

$$\frac{V_x}{R_3 // R_4} = \frac{V_{out} - V_x}{R_1} \tag{12}$$

Combine (11) and (12),

$$\frac{V_{out}}{V_{in}} = -\frac{R_3}{R_2}\frac{R_1 + R_3//R_4}{R_3//R_4} \tag{13}$$

If $R_1 \to \infty$, (13) reduces to,

$$\frac{V_{out}}{V_{in}} = -\frac{R_3}{R_2}, \tag{14}$$

which is the typical case of inverting op amp.

**Q4** Calculate the transfer function (in other word, gain) of the circuit shown in Fig. 3 if $A_0 = \infty$. Would it be possible thats $|\frac{V_{out}}{V_{in}}| = 1$ for all frequencies.

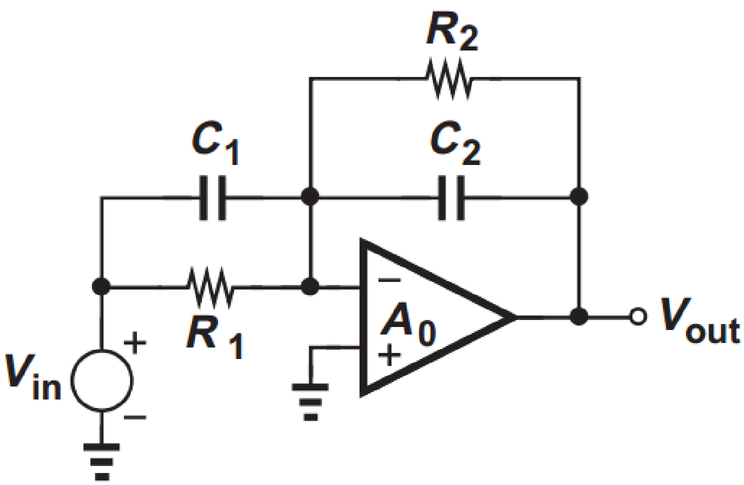


Figure 3: Sample Differentiator

**A4** Since $A_0 = \infty$, $V_- = V_+$, applying KCL,

$$\frac{V_{in}}{R_1//\frac{1}{C_1 s}} = -\frac{V_{out}}{R_2//\frac{1}{C_2 s}} \tag{15}$$

$$A(s) = \frac{V_{out}}{V_{in}} = -\frac{R_2//\frac{1}{C_2 s}}{R_1//\frac{1}{C_1 s}} \tag{16}$$

To make sure $|A(s)|$ is unity for all frequencies, thus Eq (21) should contain no frequency components, $R_1 = R_2$ and $C_1 = C_2$ is a feasible solution.

**Q5** Repeat Q4 when $A_0$ is finite.

**A5** Applying KCL, we have,

$$\frac{V_{in} - V_-}{R_1//\frac{1}{C_1 s}} = \frac{V_{out} - V_-}{R_2//\frac{1}{C_2 s}} \tag{17}$$

In addition,

$$V_{out} = -A_0 V_- \tag{18}$$

Solve (17) and (18),

$$\frac{V_{out}}{V_{in}} = -\frac{R_2//\frac{1}{C_2 s}}{R_1//\frac{1}{C_1 s}} \frac{A_0}{A_0 + 1 + R_2//\frac{1}{C_2 s}} \tag{19}$$

Let $|\frac{V_{out}}{V_{in}}| = 1$, we have,

$$|\frac{R_1//\frac{1}{C_1 s}}{R_2//\frac{1}{C_2 s}}| = \frac{A_0 + 1}{A_0 - 1} \tag{20}$$

Since $A_0$ is rational, we can still have the following result,

$$R_1 = \frac{A_0 + 1}{A_0 - 1} R_2 \tag{21}$$

and,

$$C_2 = \frac{A_0 + 1}{A_0 - 1} C_1 \tag{22}$$

**Q6** Consider the voltage adder shown in Fig. 4, where $V_1 = V_0 \sin\omega t$ and $V_2 = V_0 \sin 3\omega t$. Assume $R_1 = R_2$ and $A_0 = \infty$. Plot $V_{out}$ as a function of time.

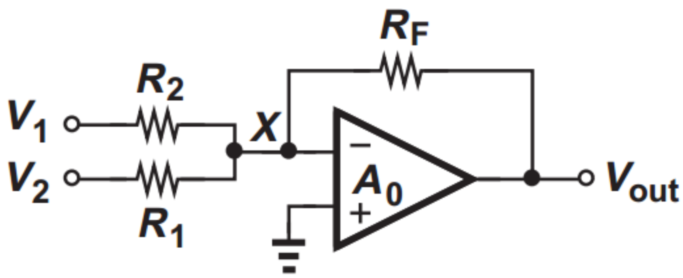


Figure 4: Voltage Adder

**A6** Obviously,

$$V_{out} = -R_F(\frac{V_1}{R_1} + \frac{V_2}{R_2}) \tag{23}$$

Since $R_1 = R_2$,

$$V_{out} = -\frac{R_F}{R_1}(V_1 + V_2) \tag{24}$$

V-t curve is shown in Fig. 5.

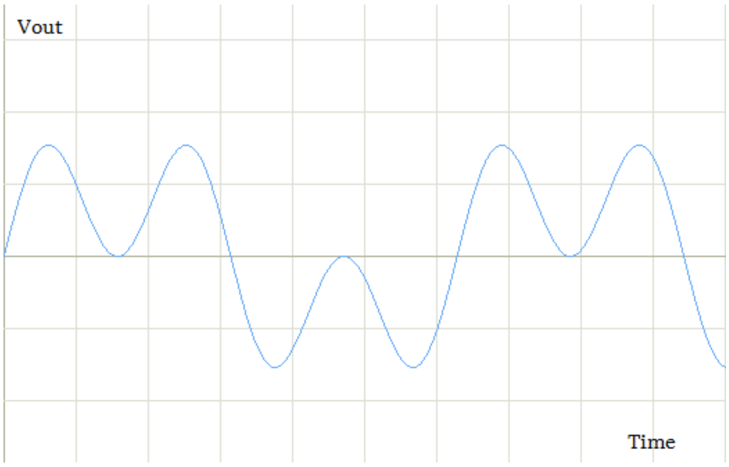


Figure 5: $V_{out}$ vs. time

**Q7** The input/output characteristic of an op amp can be approximated by the piecewise-linear behavior illustrated in Fig. 6, where the gain drops from $A_0$ to $0.8A_0$ and eventually to zero as $|V_{in1} - V_{in2}|$ increases. Suppose this op amp is used in a non-inverting amplifier (Fig. 1) with an ideal gain of 5. Plot the closed-loop input/output characteristic of the circuit.

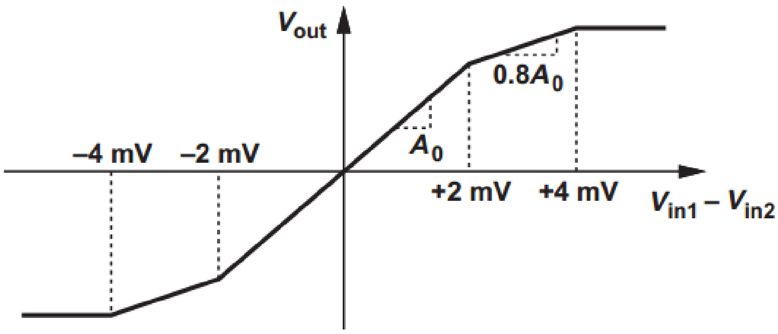


Figure 6: Open-loop Gain Variation

**A7** From Eq. (3), we have closed-loop gain,

$$g_{closed} = \frac{V_{out}}{V_{in1}} = \frac{A_0}{1 + \frac{R_2}{R_1+R_2}A_0} \tag{25}$$

Combine (4) and (25),

$$g_{closed} = 5 - \frac{25}{5 + A_0} \approx 5 - \frac{25}{A_0} \tag{26}$$

Besides,

$$V_{out} = 5V_{in2} \tag{27}$$

Then,

$$V_{in2} = (1 - \frac{5}{A_0})V_{in1}, V_{in1} - V_{in2} = \frac{5}{A_0}V_{in1} \tag{28}$$

When $V_{in1} - V_{in2}$ is $2mv$ and $4mv$, $V_{in1}$ is $\frac{A_0}{5}(2mv)$ and $\frac{A_0}{5}(4mv)$ correspondingly. Accordingly, closed-loop input/output characteristic is presented in Fig. 7.

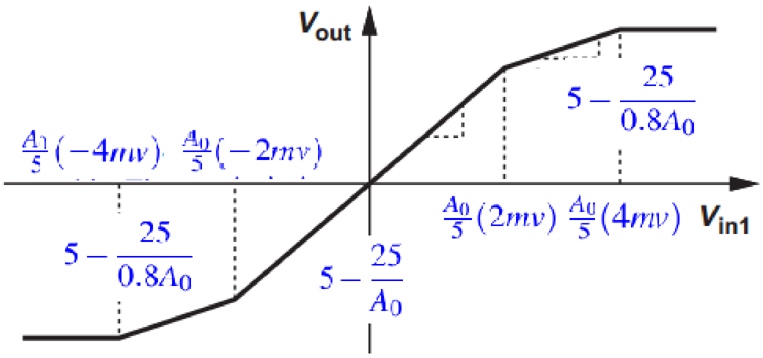


Figure 7: Closed-loop Gain Variation

**Q8** Mental-Oxide-Semiconductor-Field-Effect-Transistor (MOSFET) is the core component of a variety of amplifiers. Fig. 8 shows a common source amplifier circuit with N-type MOS (M1). Typically, when M1 works as amplifier, drain current $I_D$ has the following relationship with bias voltage $V_{in}$:

$$I_D = k(V_{in} - V_{th})^2, \quad (29)$$

where $k$ is positive and related to material properties of MOSFET and $V_{th}$ is threshold voltage to turn the device on. Calculate small-signal gain of common source amplifier and show that this amplifier is an inverting amplifier.

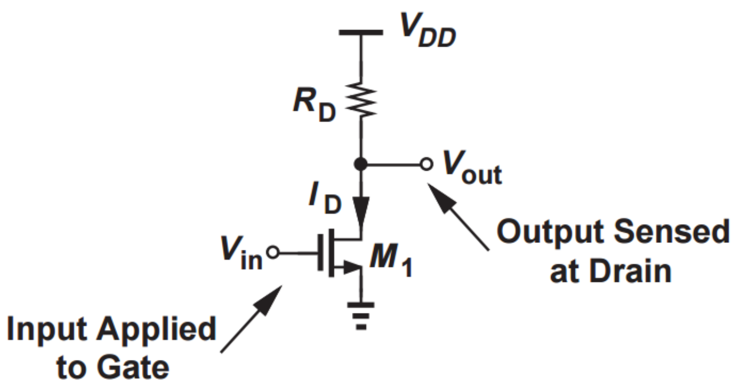


Figure 8: Common Source Amplifier

**A8** When small singal $\Delta V$ is applied on input terminal, drain current change is :

$$\Delta I_D = I'_D \Delta V_{in} \quad (30)$$

From Krichhoff's Law,

$$V_{DD} = I_D R_D + V_{out} \quad (31)$$

In addition that $V_{DD}$ is constant, i.e.

$$\Delta I_D R_D + \Delta V_{out} = 0 \tag{32}$$

Substitute (29) and (30) into (32), we can derive

$$\frac{\Delta V_{out}}{\Delta V_{in}} = -2k(V_{in} - V_{th})R_D. \tag{33}$$

Thus, gain of this amplifier is $2k(V_{in} - V_{th})R_D$, and output has a phase difference of $\pi$ from Input.