香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examinations 2006-2007

Course Code & Title : CSC5420 Computer System Performance Evaluation

Student I.D. No. : ....

1 Points Distribution

Problem	Points	Score
problem 1	10	
problem 2	10	
problem 3	10	
problem 4	10	
problem 5	10	
problem 6	20	
problem 7	20	
problem 8	10	

2 Questions

1. You are invited to attend a party in John's house. There are ten people in the party and they are randomly seated at a round table. What is the probability that you and your girlfriend will sit next to each other? Instead of ten people, what is the probability if we want to generalize it to any N people in the table.

2. Among $t = 60$ lottery tickets, $w = 20$ win prizes. We buy $b = 6$. What is the probability that $g = 2$ will be winning? Generalize this to arbitrary t, w, b, g.

3. Mrs. Lui has made a chicken and a strawberry pie for her husband and her daughter. Eating more than half of it will give indigestion to anyone. While she is away shopping, her husband helps himself to a piece of the pie. Then, the daughter comes and has a piece of what is left. When Mrs. Lui returns, she finds that more than a half of the pie is gone.

   Assuming that the size of each of the two pieces eaten is random and uniformly distributed over what is currently available, what is the probability that neither Mr. Lui and his daughter will get indigestion?

4. A router A receives Poisson packet traffic from two sources 1 and 2, at rate λ_1 and λ_2, respectively. Router A transmits packet on a FCFS basis, using a link with capacity C bit/sec. The two input streams are assumed independent and their packet lengths are identically and exponentially distributed with mean L bits. A packet from source 1 is always accepted by A. A packet from source 2 is accepted only if the number of packets in A (in queue or in transmission) is less than a given number $K > 0$; otherwise it is assumed lost.

   - What is the range of λ_1 and λ_2 for which the expected number of packets in A will stay bounded as time increases?
   - For λ_1 and λ_2 in the stable range, what is the probability of having n packets in A.
   - What is the average number and average response time for source 1?
   - What is the average number and average response time for source 2?

5. Consider a derivative $M/M/k$ system when the system empties out, it won't start any service until there are k packets in the system. Once service begins it proceeds until the system becomes empty again. What is the average response time for this derivative $M/M/1$ system?

6. Consider a *lazy* M/G/1 queueing system with arrival rate of λ in which the server goes on vacation whenever the system becomes empty. A queue of arriving customers will form while the server is on vacation, but, as soon as that queue grows to N customers , the server will immediately return and begin serving customers. Let $B^*(s)$ be the service time transform. Further, let $G^*(s)$ be the busy period transform for an ordinary M/G/1 system with no vacation and let $G_L^*(s)$ be the busy period transform for the lazy M/G/1 system with vacation.

   (a) Give the equation for $G^*(s)$.

   (b) For what value of N does the lazy M/G/1 becomes ordinary M/G/1?

   (c) Find $G_L^*(s)$ in terms of $B^*(s)$, $G^*(s)$, λ and N.

   (d) Express $G_L^*(s)$ in terms of $G^*(s)$ and N only.

(e) Suppose now that N is a random number with $p_n = P[N = n]$ with z-transform:

$$P(z) = \sum_{n=1}^{\infty} p_n z^n$$

Express $G_L^*(s)$ in terms of $P(z)$ and $G^*(s)$.

7. Assume we have an $M/G/1$ queueing system in which a departing customer leaves the system with probability $1 - p$ or re-enters service immediately with probability p (in which case a *new* service time is selected from $b(x)$ independently). Let the arrival rate be λ. Let $b_T(x)$ be the probability density function for a customer's *total* service time.

   (a) Find the mean (or first moment) of the *total* service time.

   (b) Find $B_T^*(s)$, the Laplace transform of $b_T(x)$, in terms of p and $B^*(s)$.

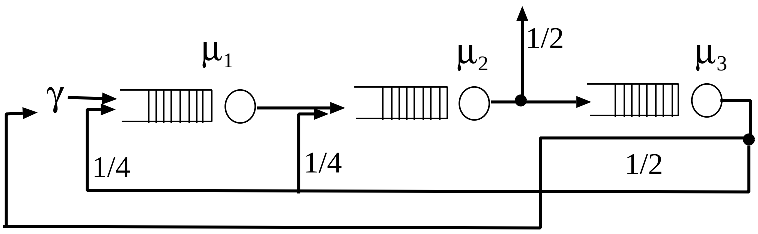
   (c) In terms of the known quantities, find $W^*(s)$, the Laplace transform of the waiting time's probability density function.

   (d) In terms of known quantities, find $Q(Z)$, the Z-transform of the number of customers in the system.

   (e) How is ρ related to the basic system parameters?

   (f) Suppose now that instead of returning to service immediately (with probability p), the customer returns to the back of the queue. Which, if any, of the answers to part (a)-(e) will change? Give only yes/no answers for this question.

8. Consider a Jackson network as shown below. Each node is a single server node with $\mu_1 =$



$\mu_2 = \mu$ and $\mu_3 = 2\mu$. The branch labels are transition probabilities.

   (a) Find λ_i for $i = 1, 2, 3$.

   (b) Express $p(n_1, n_2, n_3)$ explicitly in terms of γ and μ.

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