Examples and Applications of Binary Search

CSCI2100 Tutorial 1

Shangqi Lu

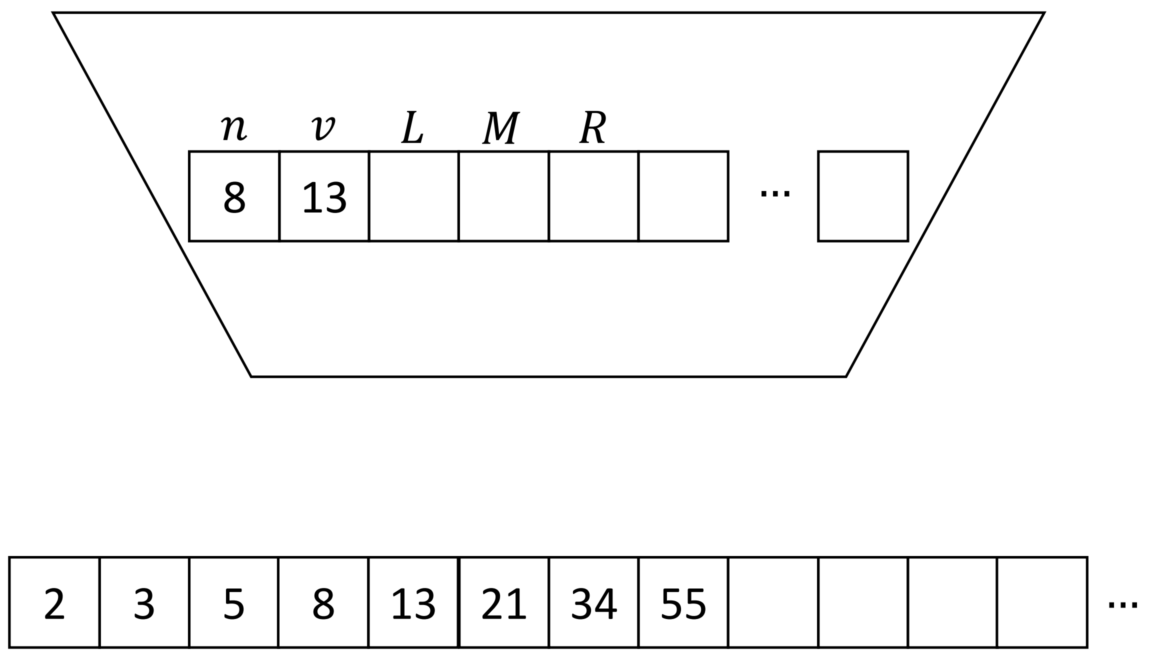
Adapted from the slides of the previous offerings of the course

Outline

- We will first review the binary search algorithm through an example
- And then use the algorithm to solve a "two-sum" problem.

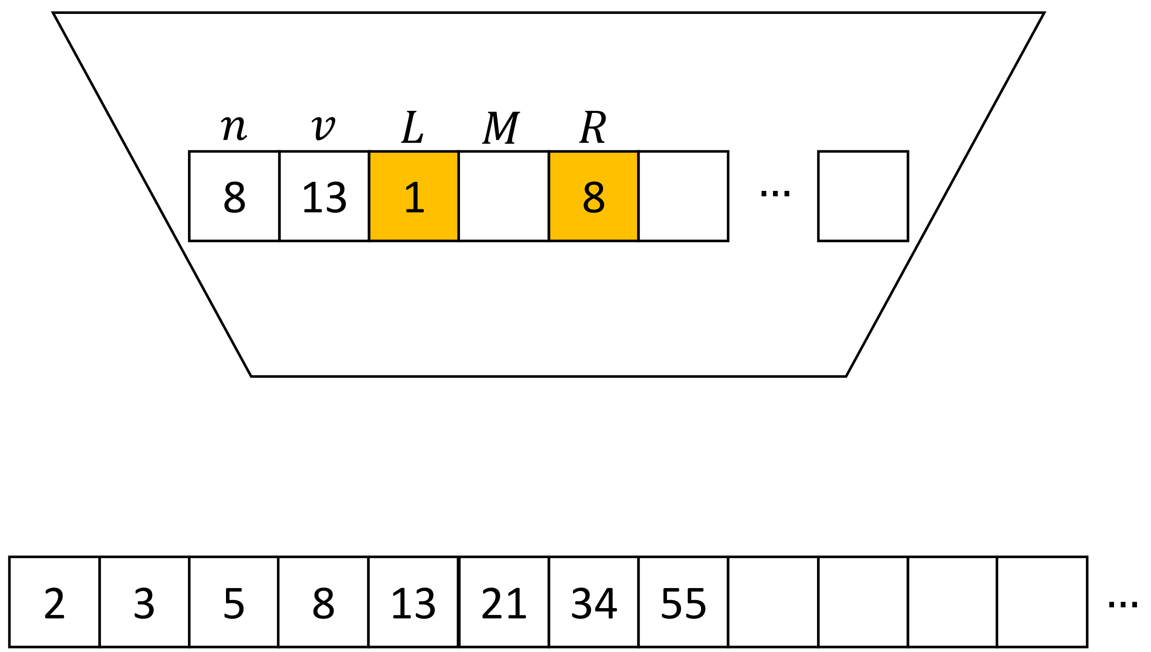
Binary Search Review

- Suppose we have the following sorted input set S, and are trying to find the value 13.



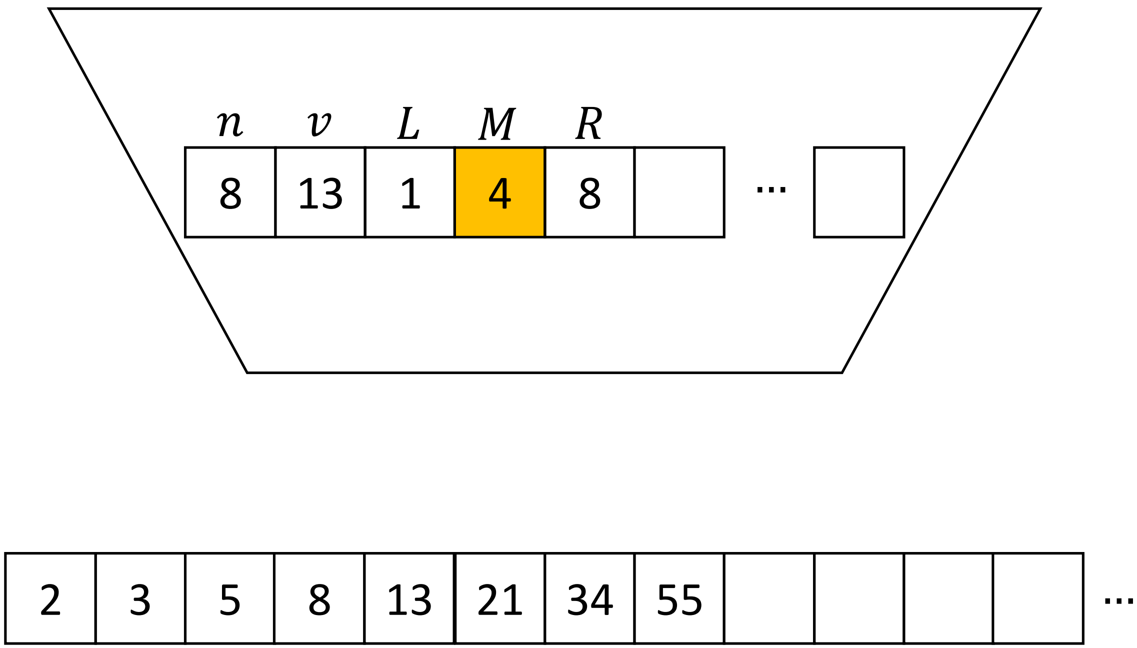
Binary Search Review

- Initializing L to be 1 and R to n (in this case, 8)



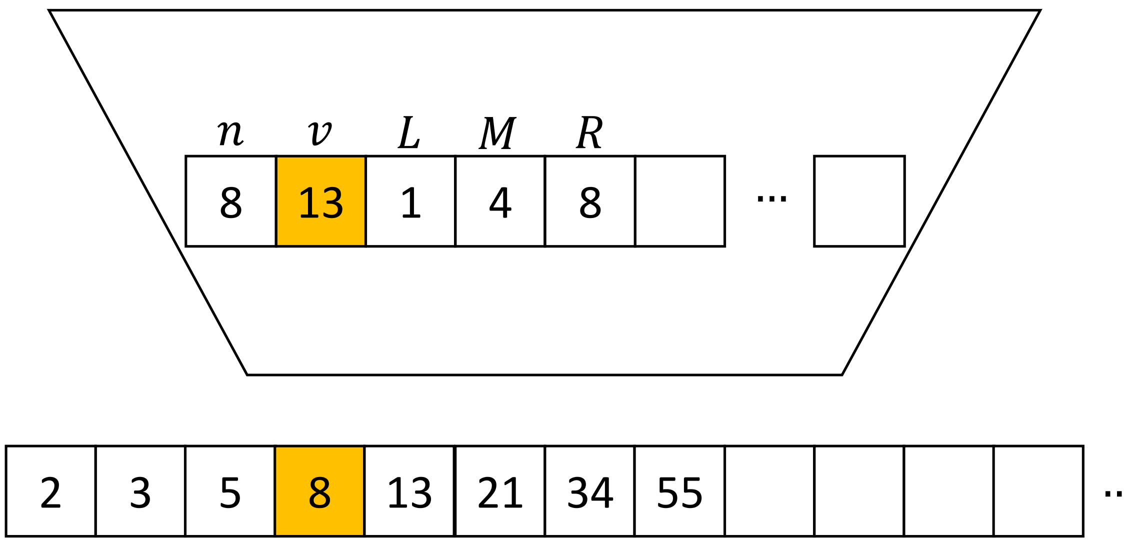
Binary Search Review

- Since $L \leq R$
- Proceed by computing $M = (L + R)/2$



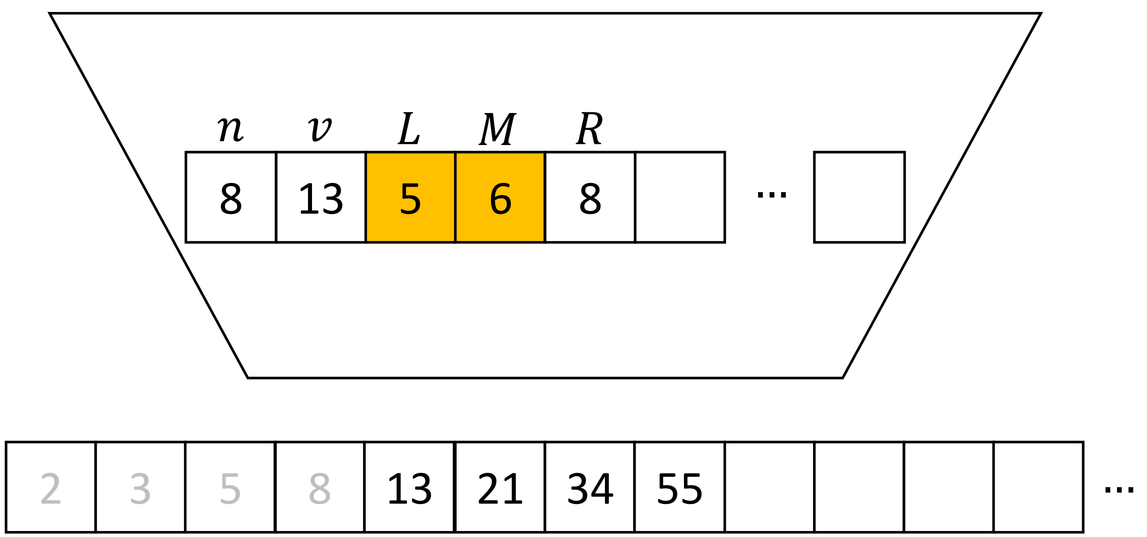
Binary Search Review

- Compare v = 13 and the value 8 indexed by M
- $v >$ the value indexed by M
- Means that the target is in the right half of the sorted sequence



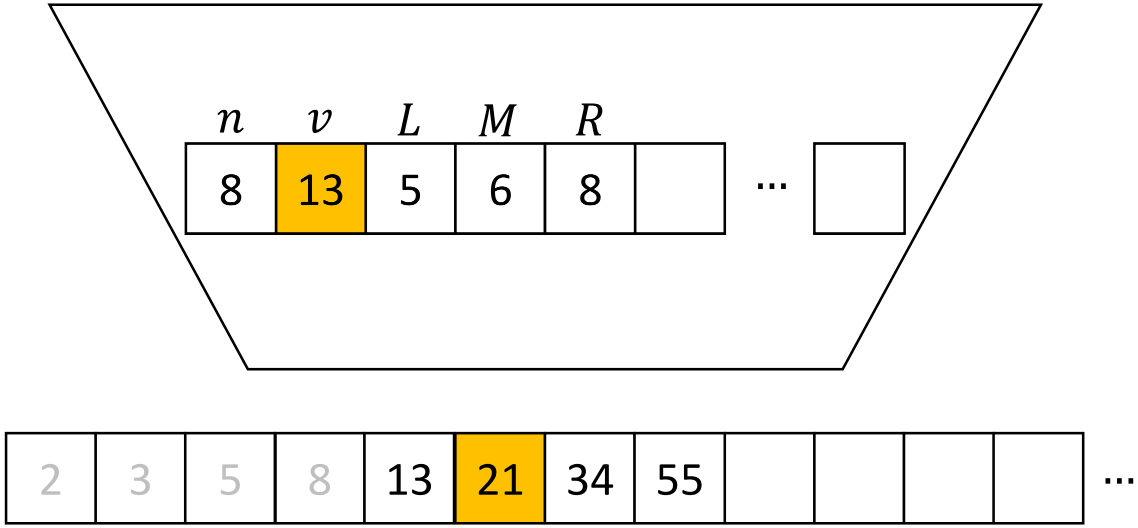
Binary Search Review

- Look at the right half of the sorted sequence
- Set L to be $M + 1$ (discard the left half)
- Recompute M



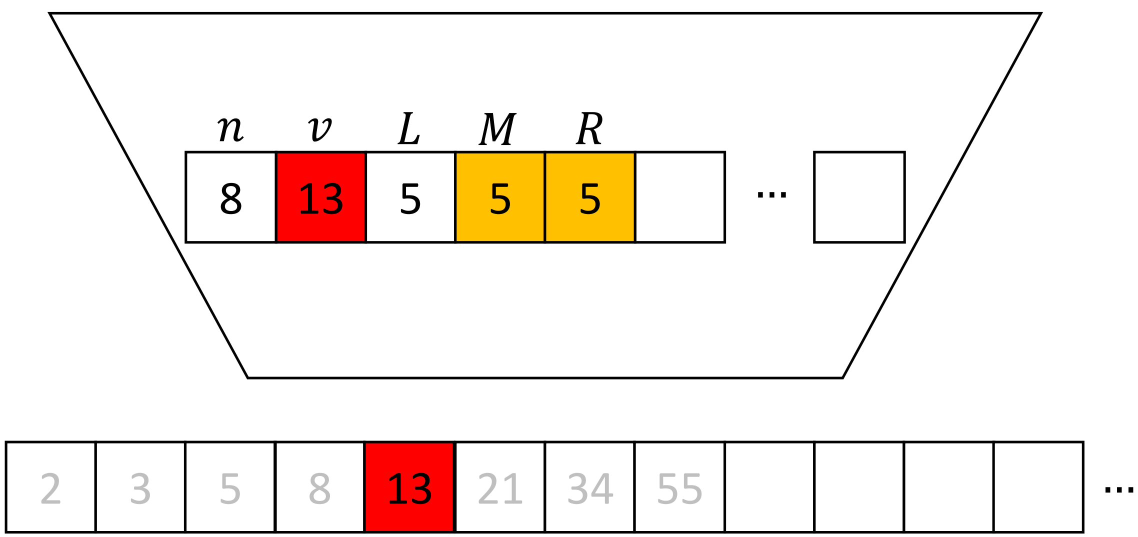
Binary Search Review

- Compare v and the value 21 indexed by M
- $v <$ the value indexed by M
- Means that the target is in the left half of the sorted sequence



Binary Search Review

- Set R to be $M - 1$ (discard the right half)
- $L, R, M = 5$
- $v =$ the value indexed by M, return “yes”



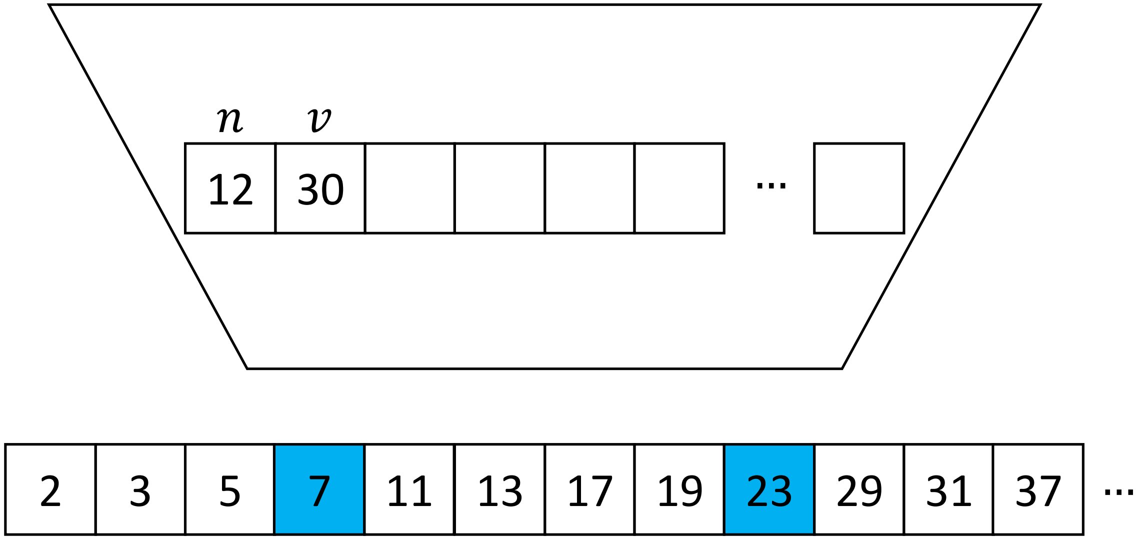
The Two-Sum Problem

- Problem Input:
  - A sequence of n positive integers in strictly increasing order in memory at the cells numbered from 1 up to n
  - The value n has been placed in Register 1
  - A positive integer v has been placed in Register 2
- Goal:
  - Determine whether if there exist two different integers x and y in the sorted sequence such that $x + y = v$

2	3	5	7	11	13	17	19	23	29	31	37	⋯

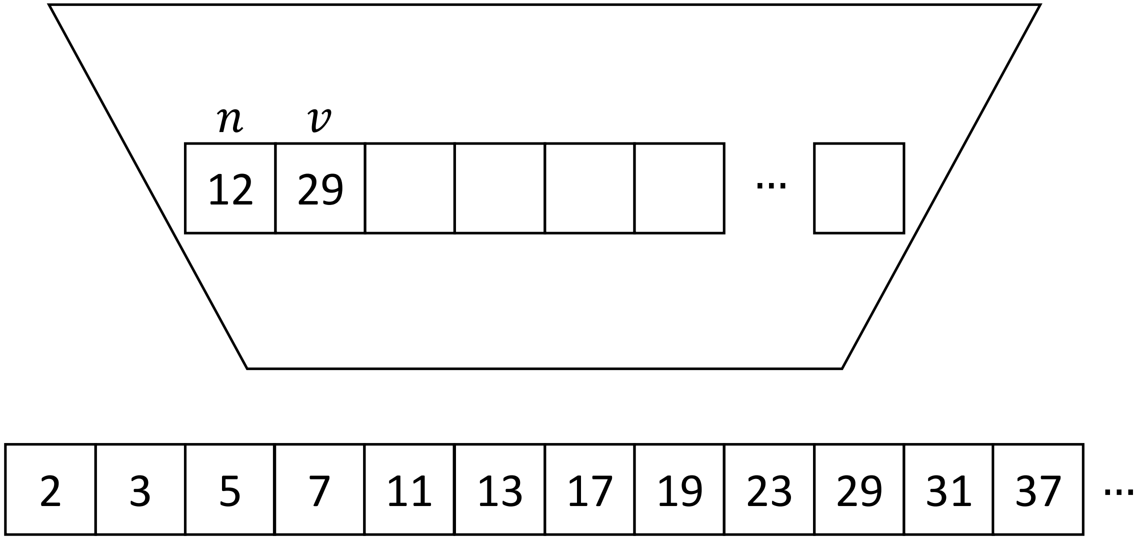
Example

- A “yes”-input with $n = 12$, $v = 30$



Example

- A "no"-input with $n = 12$, $v = 29$



A First Attempt

- Naïve algorithm:
  - Enumerate all possible pairs in the sorted sequence
  - Check if they sum to v
  - There are $\binom{n}{2} = \frac{n(n-1)}{2}$ possible pairs
  - Worst-case time: at least $n(n-1)/2$
- Can we do better than this?
- Hint: Take advantage of the fact that the given sequence is sorted!

Binary Search the Answer

- Goal: Find $a\ pair(x, y)$ such that $x + y = v$
- Observe that given x, $y = v - x$, is determined
- Improve the naïve algorithm
  - Instead of enumerating all possible y , we can find if there exits an integer $v - x$ in the sequence
- Solution:
  - For each x in the sequence:
    - set y as $v - x$
    - Use binary search to see if y exists in the sequence

The Repeated Binary Search Algorithm

- Pseudocode:

```
Let n be register 1 and v be register 2
register i ← 1, register one ← 1
while i ≤ n
    read into register x the memory cell at address i
    y ← v − x
    if BinarySearch(y) = "yes"
        return "yes"
    i ← i + one (effectively increasing i by 1)
return "no"
```

Worst-Case Running Time

- Worst case (when the output is “no”)
- This algorithm needs to run binary search n times
- Cost of each binary search: at most $10(1 + \log_2 n)$
- Cost of the algorithm: at most $100n(1 + \log_2 n)$ (a loose upper bound)

- Can we do even better?
- Actually this problem can be solved in at most $100n$ time --- left for you to try outside the class.