CSCI2100: Regular Exercise Set 1

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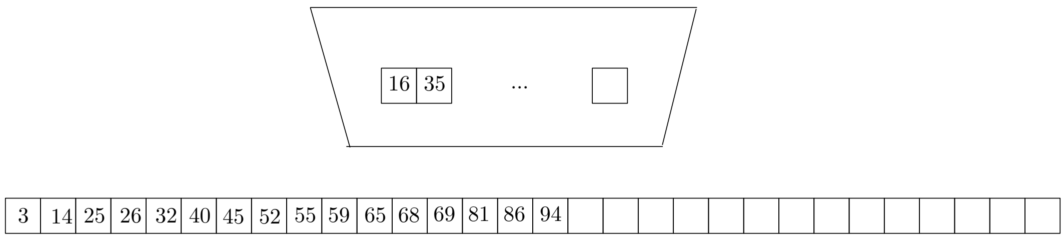
Problem 1. Let x be a real value. Define $\lfloor x \rfloor$ to be the largest integer that does not exceed x. For example, $\lfloor 2.5 \rfloor = 2$, whereas $\lfloor 3 \rfloor = 3$.

Suppose that you are given an integer $n \geq 2$ in (a register of) the CPU. Write an algorithm to compute the value of $\lfloor \log_2 n \rfloor$ in no more than $100 \log_2 n$ time.

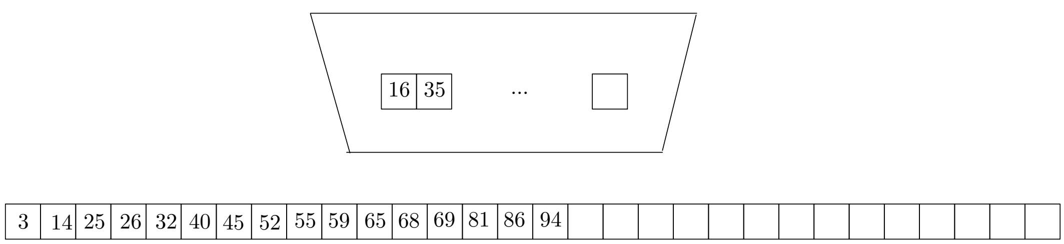
Problem 2. The following figure shows an input to the dictionary search problem.



Describe how binary search works using the input.

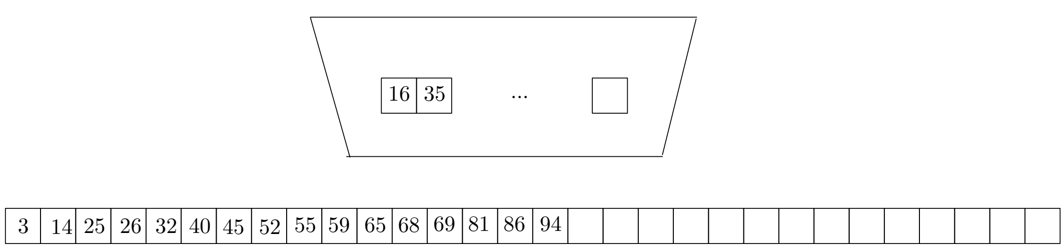
Problem 3 (Predecessor Search). Let us first define the notion of *predecessor*. Let S be a set of integers. Given an integer v, the *predecessor* of v in S is the largest integer in S that is at most v. For example, suppose $S = \{3, 14, 15, 26, 32, 40\}$. The predecessor of 25 is 15, while that of 26 is 26.

Consider the following problem. You are given a set S of n integers, which are stored at memory cells 1, 2, ..., n in ascending order. The value of n is given in the CPU, and so is an integer v. The following shows an example with $n = 16$ and $v = 35$.



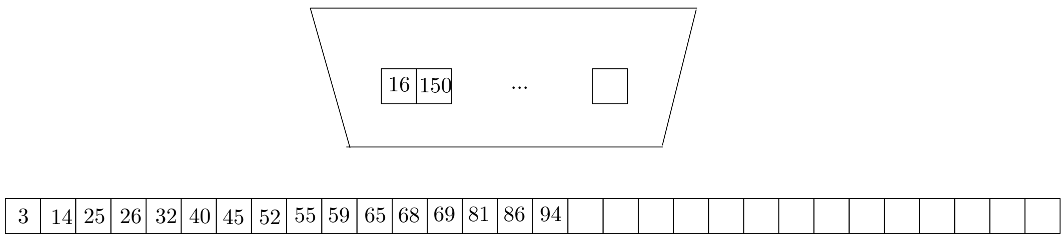
Describe an algorithm to find the predecessor of v. Your algorithm should have running time at most $100 + 100 \log_2 n$.

Problem 4 (Prefix Counting). Consider the following problem. You are given a set S of n integers, which are stored at memory cells 1, 2, ..., n in ascending order. The value of n is given in the CPU, and so is an integer v. The following shows an example with $n = 16$ and $v = 35$.



Describe an algorithm to find the number of integers in S that are at most v. In the above example, for instance, you should return 5. Your algorithm should have running time at most $100 + 100 \log_2 n$.

Problem 5 (The 3-Sum Problem). Consider the following problem. The input S consists of n integers, which are given at memory cells 1, 2, ..., n, arranged in ascending order. The value of n is given in the CPU. So is a value v. The following shows an example with $n = 16$ and $v = 150$.



Describe an algorithm to determine whether S has 3 numbers that sum up to v. In the above example, the answer is "yes" because $150 = 40 + 45 + 65$. Your algorithm should have running time at most $100 + 100 \cdot n^2 \log_2 n$.