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Eigenvector-like measures of centrality for asymmetric relations

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Abstract

Eigenvectors of adjacency matrices are useful as measures of centrality or of status. However, they are misapplied to asymmetric networks in which some positions are unchosen. For these networks, an alternative measure of centrality is suggested that equals an eigenvector when eigenvectors can be used and provides meaningfully comparable results when they cannot.

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1. Introduction to asymmetric centrality measures

Measures of network centrality exist for symmetric as well as asymmetric relations. Consider, for example, the networks as shown in Fig. 1. Depending on the content of the relationship, these networks are meant to represent that various measures of centrality can be applied (Wasserman and Faust, 1994, pp. 169–219). If these are friendship choices and popularity is being assessed in-degree, then the total number of choices received by each individual can be used as a measure. In network II, position A is the most central as measured by in-degree. If network I were a communications network we could apply betweenness centrality; by the conventional measure of betweenness centrality, position 3 is the most central, followed by position 4.

Wasserman and Faust also discuss what they call prestige measures of centrality, measures in which the centralities or statuses of positions are recursively related to the centralities or statuses of the positions to which they are connected. This is the type of centrality measure

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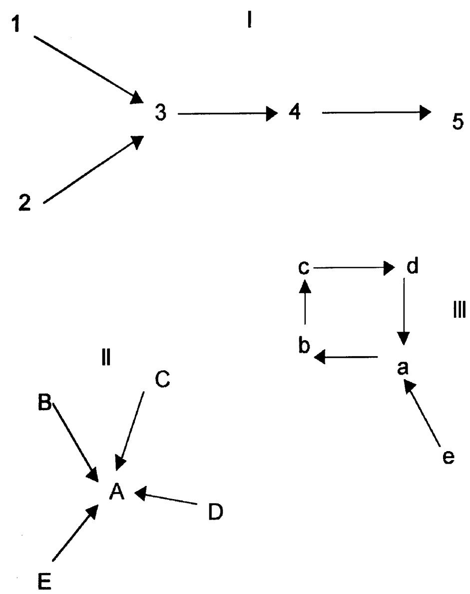

Fig. 1. Hypothetical networks.

that will be the focus of this paper. Being chosen by a popular individual should add more to one's popularity. Being nominated as powerful by someone seen by others as powerful should contribute more to one's perceived power. Having power over someone who in turn has power over others makes one more powerful. Such measures have a general form. Let A be an adjacency matrix where a_{ij} means that i contributes to j's status and let x be a vector of centrality scores. The most general form is as follows:

$$x_i = a_{1i}x_1 + a_{2i}x_2 + \cdots + a_{ni}x_n \tag{1}$$

The centrality or status of an individual is a function of the status of those who choose him.[1] The set of equations defined by (1) have a matrix representation (where A^{T} is the transpose of A).

$$A^{\mathrm{T}}x = x \tag{2}$$

The status of an actor is a linear function of the actors to whom he is connected. Such measures are easily interpretable. In a community power study, an actor's status is increased more by nominations from those who themselves have received many nominations. In a school, a student's popularity is increased more by receiving choices from students who are themselves popular. In a communications network, those who are themselves receiving

[1] In this paper, the convention will be that columns of A will always refer to those whose status is being determined by their connections to the row actors. If the data refer to nominations of who has power, $a_{ij} = 1$ if i nominates j. If the data refer to who influences whom, $a_{ij} = 1$ if i is influenced by j.

many communications from others will themselves be better and more valuable sources of information.

2. Problems with asymmetric centrality measures: two solutions

2.1. Solution 1: eigenvector centrality

In Eq. (2), x is an eigenvector of A corresponding to an eigenvalue of 1. Eq. (2) has, in general, no non-zero solution unless A has an eigenvalue of 1. One way to make the equations solvable is to normalize the rows so that each adds up to 1.[2] Then Eq. (2) has a solution because A has an eigenvalue of 1. The other way, first suggested by Bonacich (1972), is to generalize Eq. (2) so that it becomes the general eigenvector equation. The assumption then becomes that each individual's status is merely proportional (not necessarily equal) to the weighted sum of the individuals to whom she is connected. We replace Eq. (1) with Eqs. (2) and (3) with Eq. (4), which always has a non-zero solution.

$$\lambda x_i = a_{1i}x_1 + a_{2i}x_2 + \cdots + a_{ni}x_n \tag{3}$$

$$A^{\mathrm{T}}x = \lambda x \tag{4}$$

If A is an $n \times n$ matrix, Eq. (4) has n solutions corresponding to n values of λ. The general solution can then be expressed as a matrix equation, in which X is an $n \times n$ matrix whose columns are the eigenvectors of A and λ is a diagonal matrix of eigenvalues.

$$A^{\mathrm{T}}X = X\lambda \tag{4a}$$

The eigenvector has become one of the standard measures of network centrality, but none of these equations produce meaningful results for the networks in Fig. 1.[3] Positions that receive no choices have no status and contribute nothing to any other position's status. Thus, all positions in networks I and II of Fig. 1 necessarily have zero status. In network III, positions a, b, c, and d have equal status despite a's greater in-degree because e has no status to contribute.

2.2. Solution 2: alpha-centrality

A solution to this problem would be to allow every individual some status that does not depend on his or her connection to others. For example, each student in a class has some popularity that depends on his or her external status characteristics. In a communication network, each individual has sources of information that are independent of other group members. Letting e be a vector of these exogenous sources of status or information, we could replace the equations above with a new equation.

$$x = \alpha A^{\mathrm{T}}x + e \tag{5}$$

[2] Wasserman and Faust (1994, p. 207) are in error on this point. They suggest that the columns should be normalized to have a sum of one. In this case, Eq. (4) has the trivial solution of a vector of ones.

[3] For some recent examples, see Poulin et al. (2000), Kang and Choi (1999), Mizruchi and Potts (1998), Yasuda and Tokuraku (1999), Braun (1997), and Bell et al. (1999).

The parameter α reflects the relative importance of endogenous versus exogenous factors in the determination of centrality. This has the matrix solution:

$$x = (I - \alpha A^{\mathrm{T}})^{-1} e \tag{6}$$

Although it would be possible for the vector e to reflect the effects of external status characteristics, this paper is focused on the effects of network structure; e will be assumed to be a vector of ones.

For the networks in Fig. 1, the solutions to $x = (I - \alpha A)^{-1} e$ are:

$$x = \begin{pmatrix} 1 \\ 1 \\ 1+2\alpha \\ 1+\alpha+2\alpha^2 \\ 1+\alpha+\alpha^2+2\alpha^3 \end{pmatrix} x = \begin{pmatrix} 1+4\alpha \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} x = \frac{1}{1-\alpha^4} \begin{pmatrix} 1+2\alpha+\alpha^2+\alpha^3 \\ 1+\alpha+2\alpha^2+\alpha^3 \\ 1+\alpha+\alpha^2+2\alpha^3 \\ 1+\alpha+\alpha^2+\alpha^3+\alpha^4 \\ 1 \end{pmatrix} \tag{6a}$$

Note that in these simple cases, the measures do exactly what we want. In network II, position A is the most central for any value of $\alpha > 0$. In network III, the order is $x_a > x_b > x_c > x_d > x_e$ for any $\alpha > 0$. In network I the results are conditional on the value of α. When $\alpha < 1/2$, $x_3 > x_4 > x_5$. For $\alpha > 1/2$, $x_3 < x_4 < x_5$. Status rankings should depend on the degree to which status is transferable from one person to another.

This measure of centrality is almost identical to a measure proposed by Katz (1953). Katz suggested that influence could be measured by a weighted sum of all the powers of the adjacency matrix A. Powers of A give indirect paths connecting points. Giving higher powers of A less weight would index the attenuation of influence through longer paths. In particular, Katz suggests that we measure influence by the column sums of the following matrix, where a is an "attenuation" factor.

$$\sum_{i=1}^{\infty} a^i A^i \tag{7}$$

This infinite sum converges to $(I - aA)^{-1} - I$ as long as $|a| < 1/\lambda_1$, where λ_1 is the maximum value of an eigenvalue of A. A little algebraic re-arrangement shows that (7) is (almost) identical to (6). If we let e be a vector of ones (*reflecting* equal exogenous status for all actors), then the column sums of the matrix in (7) equals its transpose times e.

$$\left(\sum_{i=1}^{\infty} a^i A^{\mathrm{T}i}\right) e = \left(-I + \sum_{i=0}^{\infty} a^i A^{\mathrm{T}i}\right) e = (-I + (I - aA^{\mathrm{T}})^{-1}) e = -e + x$$

Eqs. (6) and (7) differ only by a constant (one in this case). Katz's approach is identical to the exogenous status model in Eq. (6).

Hubbell (1965) also proposed a slightly different version of this model. Hubbell's equation is $S = E + WS$, where E is a vector of exogenously determined statuses, S the vector of status in a group, and W the matrix of nominations whose rows are normalized.[4] Finally,

[4] The normalization is that every row adds up to a sum ≤ 1.00 and one row sum is strictly <1.00.

in order to create a general model that would be consistent with the experimental results for negatively connected exchange networks Bonacich (1987) proposed a model formally identical to the Katz model with β substituted for Katz's a.[5]

3. The relation between the two solutions

Eq. (5) was introduced to handle problems in using eigenvectors in asymmetric matrices, but it turns out to be almost identical to the eigenvector measure for symmetric matrices. Thus, it is a close generalization of the eigenvector measure. Let $\{x_i\}$ be the set of orthonormal eigenvectors of an $n \times n$ symmetric matrix A and let $\{\lambda_i\}$ be the associated set of eigenvalues.[6] Any symmetric matrix can be decomposed as follows:

$$AX = X\lambda \quad \text{and} \quad X^{\mathrm{T}} = X^{-1}, \quad \text{therefore,} \quad A = X\lambda X^{-1} = X\lambda X^{\mathrm{T}}, \quad \text{or}$$

$$A = \sum_{i=1}^{n} \lambda_i x_i x_i^{\mathrm{T}} \tag{8}$$

Because the eigenvectors of a symmetric matrix are orthogonal, powers of A have similar decompositions.

$$A^k = \sum_{i=1}^{n} \lambda_i^k x_i x_i^{\mathrm{T}} \tag{9}$$

Now let us re-examine x in terms of the eigenvectors of $A = A^{\mathrm{T}}$. We will continue to assume that $\alpha < 1/\lambda_1$.

$$x = (I - \alpha A)^{-1} e = \left(\sum_{k=0}^{\infty} \alpha^k A^k \right) e = \left(\sum_{k=0}^{\infty} \alpha^k \sum_{i=1}^{n} \lambda_i^k x_i x_i^{\mathrm{T}} \right) e$$
$$= \left(\sum_{i=1}^{n} \left(\sum_{k=0}^{\infty} \alpha^k \lambda_i^k \right) x_i x_i^{\mathrm{T}} \right) e = \sum_{i=1}^{n} \frac{1}{1 - \alpha\lambda_i} x_i x_i^{\mathrm{T}} e \tag{10}$$

Now we will assume that λ_1 is strictly greater than any other eigenvalue.[7] As α approaches $1/\lambda_1$ the coefficients for all eigenvectors after the first shrink in importance and in the limit x is proportional to x_1.[8]

$$\lim_{\alpha \to (1/\lambda_1)-} (1 - \alpha\lambda_1)(x) = (x_1^{\mathrm{T}} e) x_1 \tag{11}$$

[5] Katz's a is always positive while Bonacich's β can be positive or negative. The point of the Bonacich (1987) paper was to create a centrality measure for exchange networks, in which β is negative. The purpose of the current paper, however, is to show that a similar approach yields an eigenvector-based centrality measure for asymmetric relations.

[6] Every symmetric matrix has a set of orthogonal eigenvectors.

[7] This will be true for almost all data matrices. Without this assumption other eigenvector-based measures, such as the principal component and the highest canonical correlation, for example, cannot be determined.

[8] Actually, α should approach $1/\lambda_1$ from below; otherwise the infinite series does not converge. And of course, when $\alpha = 1/\lambda_1$ the series is undefined and $(I - \alpha A)$ is singular.

Thus, for a symmetric matrix the measure of alpha-centrality converges to the standard eigenvector measure of centrality.[9]

There is another model of status that also resolves into an eigenvector. Suppose that, as a first approximation, each individual is given an equal status, represented by the vector e of ones. We first count the number of nominations of status or power received from others as recorded in an adjacency matrix A. This is the vector Ae. To improve this measure we weight each individual's nomination of others by the nominations he received; this is $A(Ae) = A^2e$. We could then weight the choices by this new vector to produce a revised measure A^3e. The divergence of this sequence could be prevented by dividing at each stage by the largest eigenvalue λ_1.[10] At the kth iteration we have

$$\frac{A^k e}{\lambda_1^k} = \sum_{i=1}^{n} \left(\frac{\lambda_i}{\lambda_1}\right)^k x_i^{\mathrm{T}} e x_i \tag{12}$$

If λ_1 is strictly greater than any other eigenvalue the ratio in parentheses approaches zero for all but the first eigenvalue, and the left-hand side of Eq. (12) approaches a multiple of x_1.

4. Asymmetric adjacency matrices

The eigenvectors of asymmetric matrices are not orthogonal, so the equations are a bit different. For an asymmetric matrix $A^{\mathrm{T}}X = X\lambda$ as before, but $X^{\mathrm{T}} \neq X^{-1}$. However, it is true that $A^{\mathrm{T}k} = X\lambda^k X^{-1}$. Letting y_i be the ith row of X^{-1},

$$A^{\mathrm{T}} = \sum_{i=1}^{n} \lambda_i x_i y_i \text{ and } A^{\mathrm{T}k} = \sum_{i=1}^{n} \lambda_i^k x_i y_i \tag{13}$$

Therefore,

$$x = (I - \alpha A^{\mathrm{T}})^{-1} e = \left(\sum_{k=0}^{\infty} \alpha^k A^{k\mathrm{T}}\right) e = \left(\sum_{k=0}^{\infty} \alpha^k \sum_{i=1}^{n} \lambda_i^k x_i y_i\right) e$$
$$= \left(\sum_{i=1}^{n} \left(\sum_{k=0}^{\infty} \alpha^k \lambda_i^k\right) x_i y_i\right) e = \sum_{i=1}^{n} \frac{y_i e}{1 - \alpha\lambda_i} x_i \tag{14}$$

If λ_1 is strictly greater than any other eigenvalue the coefficient for the first term in the final sum in (14) will become more and more dominant as α approaches $1/\lambda_1$. Therefore, we can say that

$$\lim_{\alpha \to (1/\lambda_1)-} x(1 - \alpha\lambda_1) = (y_1 e) x_1 \tag{15}$$

[9] Although the exogenous status measure has the eigenvector as a limit, it is never equal to the eigenvector for any value of α. The usefulness of a family $x(\alpha)$ of centrality measures generated by different values of α is not discussed in this paper.

[10] For simplicity, we will continue to assume that there is a single largest eigenvalue. This is almost certainly true for any real data.

The eigenvector and alpha-centrality approaches are identical (as α approaches the limit $1/\lambda_1$ and if there is a largest eigenvalue) even for asymmetric matrices. However, this approach does not work well or at all for asymmetric networks like those in Fig. 1. Positions not receiving nominations contribute nothing to the status of others. However, the alpha-centrality approach can still be used. The solutions in (6a) for the networks in Fig. 1 are a reasonable extrapolation of the eigenvector approach to networks in which eigenvectors should not be used.

5. The Friedkin–Johnsen model

Friedkin and Johnsen (1997) propose a dynamic model of influence in which attitudes at time t, y_t, are a function of attitudes at time $t-1$. W is a normalized adjacency matrix, D is a diagonal matrix representing the degree to which each individual's attitudes are determined by others, and y_0 is the individual's initial attitudes. [11]

$$y_t = DWy_{t-1} + (I - D)y_0 \tag{16}$$

At equilibrium $y_t = y_{t-1} = y$, and solving for y:

$$y = (I - DW)^{-1}(I - D)y_0 \tag{17}$$

The Friedkin–Johnsen model (17) and the alpha-centrality model (5) could be applied to exactly the same situation. Suppose that a group has a network influence structure initial attitudes described by a vector y_0. In this situation, one could either assess power or attitude change. The Friedkin–Johnsen model would aim to explain the distribution of attitudes y and the alpha-centrality model would attempt to measure power x. Here are some of the similarities and differences between the models:

1. Both A in (5) and W in (17) describe the pattern of influence in a group.
2. W is a function of A. [12] Let a_{ij} mean that i is influenced by j. W is A normalized so that each row sums to 1, and w_{ij} is the proportion of the total group influence on i that comes from j.
3. Attitude y is a function of W while x is a function of A^{T}.
4. The diagonal matrix D and the scalar α both reflect the degree to which individuals in the group are endogenously influenced. Their meanings are somewhat different; D reflects the degree to which attitudes are endogenously influenced, whereas α reflects the degree to which group members' power or status is endogenously influenced. Moreover, the D in (17) allows individuals to have different degrees of endogenous susceptibility, while the scalar α presumes that all individuals have the same susceptibility.
5. The entity e in (5) is a vector of exogenous power or status, while y_0 is a vector of initial attitudes.

[11] Friedkin and Johnsen use the symbol A to refer to what we have called D in their model. We have made the change to avoid confusion with my use of A to refer to an adjacency matrix.

[12] A is also a function of W.

We can compare the models by seeking a common ground. Suppose that $D = \delta I$, reflecting the assumption of a common susceptibility to influence. Let $A = W$; nothing in the mathematics prevents A in (5) from being normalized in this way. Finally, let us assume for convenience that $\alpha = \delta$; the types of susceptibility are equal. What remains as a key difference is that x is a function of W^{T} (namely, $x = \alpha A^{\mathrm{T}} x + e = \delta W^{\mathrm{T}} x + e$) while y is a function of W (namely, $y = DWy + (I - D)y_0 = \delta Wy + (1 - \delta)y_0$). The two models can be identified by recognizing that the alpha-centrality model (5) deals with the column sums of the matrix $(I - \delta W)^{-1}$ whose rows determine attitudes in (17).

$$\frac{y}{(1-\delta)} = (I - \delta W)^{-1} y_0, \quad x = (I - \delta W^{\mathrm{T}})^{-1} e \tag{18}$$

Having seen the essential similarities we can add back to the Friedkin–Johnsen model the assumption that susceptibilities vary. This suggests a new utility for the Friedkin–Johnsen model: *the column sums of* $(I - DW)^{-1}$ *are a measure of power*.

6. An example: Padgett's Florentine marriage data

Padgett and Ansell (1993, p. 1265) have given us directional asymmetric data on marriage patterns among 26 early 15th century Florentine elite families. A directed arrow from one family to another in Fig. 2a of the study by Padgett and Ansell (1993, p. 1276) shows that a male in the former family married a female in another. If "marriage" is the tie, then the symmetric data, in which there is a relation between two families if there is a marriage tie between them, should be analyzed. Padgett and Ansell (1993, pp. 1293–1294), however, suggest that the data reflect an underlying status dimension where the sending of a male from one family to another is a sign that the latter family has higher status. The first column of scores in Table 1 shows the eigenvector centrality of the transpose of the matrix of this data: families have higher status the higher the status of the families from which they receive sons-in-law. The data have been arranged in order of the first column. The second column of scores gives the alpha-centrality status scores when $\alpha = 0.40$.[13]

Although the two columns of scores are highly correlated there are important differences. In the eigenvector list, there are 12 families that have scores of zero. Six of these 12 families received no males in marriage relations with other families. However, four of these six families have zero scores because the families they received males from received no males from other families. And, two families have scores of zero even though they received males from other families and the families they received males from also received males from yet other families. For example, a daughter from the Bischeri family married a Guadagni and a female Guadagni married a Fioravanti, yet the former two have scores of zero. The reason is that no males married into the Fioravanti family. All these anomalies are corrected in the second column. Only those families that received no males, like the Fioravantis, earn the lowest possible score of one. The Guadagni family, who receive a male from the Fioravantis, have a score of 1.4, and the Bischeris, who receive a male from a family that receives males, have a score of 1.56.

[13] $1/\lambda_1$ for this data was 0.552.

Table 1
Eigenvector and alpha-centrality status analysis of Padgett Florentine marriage data

Name	$A^{\mathrm{T}}x = \lambda x$	$(I - \alpha A^{\mathrm{T}})^{-1}e$
1. Guasconi	3.27603	5.12576
2. Albizzi	3.26387	5.15345
3. Peruzzi	2.80326	4.4415
4. Panciatichi	2.54878	4.15672
5. da Uzzano	1.80998	3.0503
6. Medici	1.80998	3.4503
7. Strozzi	1.80998	3.4503
8. Altoviti	1.80326	3.06138
9. Castellani	1.54878	2.7766
10. della Casa	1.54878	2.7766
11. Rucellai	1	2.38012
12. Tornabuoni	1	2.38012
13. Pepi	0.85569	2.11064
14. Rondinelli	0.85569	2.11064
15. Scambrilla	0.85569	2.11064
16. Benizzi	0	1
17. Bischeri	0	1.56
18. Cocco-Donati	0	1.4
19. dall'Antella	0	1
20. Davanzati	0	1
21. Dietisalvi	0	1.56
22. Fioravanti	0	1
23. Ginori	0	1.4
24. Guadagni	0	1.4
25. Guicciardini	0	1
26. Orlandini	0	1.4
27. Valori	0	1

7. Conclusions

The eigenvector captures a certain aspect of centrality or status that is not captured by other measures. The eigenvector is an appropriate measure when one believes that actors' status is determined by those with whom they are in contact. This conception of importance or centrality makes sense in a variety of circumstances. Social status rubs off on one's associates. Receiving information from knowledgeable sources adds more to one's own knowledge. However, eigenvectors can give weird and misleading results when misapplied.

We have outlined two approaches, eigenvectors and alpha-centrality. The second is always applicable regardless of the type of relation in the network while the first is only applicable to some networks. The two are equal when both apply. This provides a justification for calling the alpha-centrality approach a generalized eigenvector measure of centrality. Its identity to the eigenvector when both apply means that it can be interpreted in a parallel manner.

In our summary, we will be assuming that the network forms one weakly connected component (Wasserman and Faust, 1994, p. 133); it is impossible to partition the vertices

Table 2
Suitability of two centrality measures

	I (symmetric relations)	II (asymmetric relations in which no individual is unchosen)	III (asymmetric but not antisymmetric relations with unchosen individuals)	IV (antisymmetric relations)
$A^{T}x = \lambda x$	√	√		
$x = (I - \alpha A)^{-1} e$	√	√	√	√

without breaking connections between them. Given this assumption, there are four types of relations whose eigenvector centrality properties differ:

1. symmetric relations;
2. asymmetric relations in which no individual is unchosen;
3. asymmetric but not antisymmetric relations with unchosen individuals;
4. antisymmetric relations. [14]

These four types of relations plus the two models of centrality that we have discussed, generate the results as in Table 2.

For networks of types I, II, and III, the two solutions give identical results as α approaches $1/\lambda_1$. [15] However, in Type III networks, unchosen individuals are ignored and have no effect on the status of others. In addition, in type IV networks the two solutions are not identical because all the eigenvalues are equal. [16]

The resolution is that for networks of types I and II it makes no difference what approach is used. The eigenvector solution may be preferable because it can be computed by standard packages. For networks of types III and IV, the alpha-centrality approach should be used and can be interpreted like an eigenvector measure of centrality. [17]

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[14] A relation is antisymmetric if *aRb* and *bRa* implies $a = b$. Two different individuals cannot have the relation to each other. For example, "is taller than" is antisymmetric. The eigenvalues for the adjacency matrix of an antisymmetric relation (with zeros on the main diagonal) are all zero.

[15] Again, we assume that there is a largest eigenvalue.

[16] If the main diagonal of the adjacency matrix consists entirely of zeros, as is ordinarily the case, the eigenvalues are all equal to zero, and so the derivation in Eq. (15) does not work.

[17] This paper does not address how to choose α. For some networks the results will depend significantly on the choice of α. Large values of α mean that status is endogenously determined, while small values mean that internal group status is relatively unimportant in determining overall status. In type III networks, α should be less than but not near $1/\lambda_1$. In type IV networks, there are no limitations on α.

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