Homework # 5
Due: April 11, 2011, 4:30 PM

Instructor: John C.S. Lui

Note: Deduction Policy for Late Submissions.

Late submissions **must** be handed to TA'office (Room 120, SHB) by person. We will not handle any late submissions in the homework collection box in 10/F, SHB.

- Deduct 30% for one day late submission (within 24 hours).

- Deduct 60% for two days late submission (within 48 hours).

- Deduct 100% for more than two days late submission (after 48 hours).

1. Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

   (a) What is the value of c?
   (b) What is the cumulative distribution function of X?

2. A system consisting of one original unit plus a spare can function for a random amount of time X. If the density of X is given (in units of months) by

$$f(x) = \begin{cases} Cxe^{-x/2} & x > 0 \\ 0 & x \le 0 \end{cases} \tag{2}$$

   What is the probability that the system functions for at least 5 months?

3. A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable with probability density function

$$f(x) = \begin{cases} 5(1-x)^4 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

   What must the capacity of the tank be so that the probability of the supply's being exhausted in a given week is .01?

4. A bus travels between the two cities A and B, which are 100 miles apart. If the bus has a breakdown, the distance from the breakdown to city A has a uniform distribution over (0,100). There is a bus service station in city A, in B, and in the center of the route between A and B. It is suggested that it would be more efficient to have the three stations located 25,50, and 75 miles, respectively, from A, Do you agree? Why?

5. Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that $P\{X > c\} = .10$.

6. Suppose that the height, in inches, of a 25-year-old man is a normal random variable with parameters $\mu = 71$ and $\sigma^2 = 6.25$. What percentage of 25-year-old man are over 6 feet, 2 inches tall? What percentage of men in the 6-footer club are over 6 feet, 5 inches?

7. A model for the movement of a stock supposes that if the present price of the stock is s, then, after one period, it will be either us with probability p or ds with probability $1 - p$. Assuming that successive movements are independent, approximate the probability that the stock's price will be up at least 30 percent after the next 1000 periods if $u = 1.012$, $d = 0.990$, and $p = 0.52$.