Large deviation bounds summary

1. Markov's inequality: $P(X \geq a) \leq E[X]/a$.
   - Applies to any *non-negative* random variable X, and any $a > 0$ ($a > E[X]$ for a meaningful bound).
   - Requires only knowledge of $E[X]$
   - Generally useful when $E[X]$ is small and X is "concentrated" around $E[X]$.
2. Chebyshev's inequality: $P(|X - \mu| \geq t\sigma) \leq 1/t^2$, where $\mu = E[X], \sigma = \sqrt{\text{Var}[X]}$.
   - Applies to any random variable X (with finite μ, σ), and any $t > 0$ ($t > 1$ for a meaningful bound).
   - Requires knowledge of both $E[X]$ and $\text{Var}[X]$.
   - Can be used to bound both $P(X \geq a)$ and $P(X \leq a)$.
3. Central Limit Theorem: If $X = X_1 + \cdots + X_n$ where X_i independent and have same PDF/PMF, then $(X - E[X])/\sqrt{\text{Var}[X]} \approx \text{Normal}(0, 1)$.
   - Applies for X being sum of many *independent* random variables.
   - Requires knowledge of $\mu = E[X_i]$ and $\sigma^2 = \text{Var}[X_i]$ to obtain $E[X] = n\mu$, $\text{Var}[X] = n\sigma^2$.
   - Approximates the CDF of X, but does not provide an error on the quality of the approximation.[1] Using the axioms, we can use it to approximate probabilities of other events like $P(150 < |X| < 200)$.

[1] This error will depend on the PDF/PMF of X_i. The Berry-Esseen Theorem is a refinement of the Central Limit Theorem that gives a explicit error bound. Away from the mean, Chernoff bounds give much tighter estimates for many random variables of interest.