An apple store observes the following purchasing habits of a typical customer:

product	green apple	red apple	golden apple
price	\$2	\$5	\$10
probability of purchase	25%	20%	5%

Assuming 1000 customers will walk into the store today, use the Central Limit Theorem to estimate the probability that at least \$400 will be spent in total. Assume that each customer purchases at most one apple and that their purchases are independent.

Solution: Let $X = X_1 + \cdots + X_{1000}$ be the amount spent in the store. The random variables X_i have expectation

$$\mathrm{E}[X_i] = 2 \cdot 0.25 + 5 \cdot 0.2 + 10 \cdot 0.05 = 2$$

and variance

$$\mathrm{Var}[X_i] = \mathrm{E}[(X_i - 2)^2] = (-2)^2 \cdot 0.5 + 0^2 \cdot 0.25 + 3^2 \cdot 0.2 + 8^2 \cdot 0.05 = 7.$$

As the X_i are independent and have the same PMF, X can be approximated by a normal random variable with mean $\mathrm{E}[X] = 1000 \cdot 2 = 2000$ and variance $\mathrm{Var}[X] = 1000 \cdot 7 = 7000$. So for a Normal$(0, 1)$ random variable N,

$$\mathrm{P}[X \geq 400] \approx \mathrm{P}[2000 + N\sqrt{7000} \geq 400] \approx \mathrm{P}[N \geq -19.123] = 1 - \mathrm{P}[N < -19.123] \approx 1 - 8.02 \cdot 10^{-82}.$$

In words, the probability predicted by the Central Limit Theorem is 1 to within 81 decimal places! (For comparison, the probability that zero dollars are spent is exactly $0.5^{1000} \approx 9.33 \cdot 10^{-302}$.)