

A five person committee is chosen at random from 5 girls and 15 boys. Conditioned on there being at least one girl on the committee, what is the expected number of boys?

Solution: Let G be the (unconditional) number of girls on the committee. By the conditional expectation formula

$$E[G] = E[G|G = 0]P(G = 0) + E[G|G > 0](1 - P(G = 0)).$$

By linearity of expectation, $E[G] = 5 \cdot (5/20) = 5/4$. On the other hand, $G = 0$ occurs only when all five members are boys, so by the product formula

$$P(G = 0) = \frac{15}{20} \cdot \frac{14}{19} \cdot \frac{13}{18} \cdot \frac{12}{17} \cdot \frac{11}{16} \approx 0.194.$$

Therefore

$$E[G|G > 0] = \frac{E[G]}{1 - P(G = 0)} \approx \frac{5/4}{1 - 0.194} \approx 1.550.$$

If B is the number of boys then $B + G = 5$ so by linearity of expectation again

$$E[B|G > 0] = 5 - E[G|G > 0] \approx 3.450.$$