Alice, Bob, and Charlie each toss a 6-sided die. What is the probability that Charlie's face value is strictly larger than both Alice's and Bob's?

Solution: The sample space consists of all $6^3 = 210$ possible outcomes (a, b, c) of Alice's, Bob's, and Charlie's dice. The event E of interest consists of those outcomes in which $c > a$ and $c > b$. We can write E as a disjoint union of $E_1, E_2, \ldots, E_6$ where E_c consists of those outcomes in which Charlie's toss is a c. Then E_c is a product set of size $(c-1)^2$ as Alice's and Bob's outcomes can have arbitrary values between 1 and $c-1$. Therefore

$$|E| = |E_1| + |E_2| + \cdots + |E_6| = 0^2 + 1^2 + \cdots + 5^2 = 55,$$

so by the equally likely outcomes formula,

$$\mathrm{P}(E) = \frac{55}{216} \approx 0.255.$$