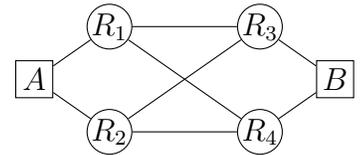


Each question is worth 10 points. Explain your answers clearly.

1. Urn A has 4 blue balls. Urn B has 1 blue ball and 3 red balls.
 - (a) You draw a ball from a random urn and it is blue. What is the probability that it came from urn A?
 - (b) You draw another ball from the same urn. What is the probability that the second ball is also blue?

2. Computers A and B are linked through routers R_1 to R_4 as in the picture. Each router fails independently with probability 10%.



- (a) What is the probability there is a connection between A and B ?
 - (b) Are the events “there is a connection between A and B ” and “exactly two routers fail” independent? Justify your answer.
3. A bus takes you from A to B in 10 minutes. On average a bus comes once every 5 minutes. A taxi takes you in 5 minutes, and on average a taxi comes once every 10 minutes. Their arrival times are independent exponential random variables. A bus comes first.
 - (a) If you want to minimize the (expected) travel time, should you take this bus?
 - (b) If you do take the bus, what is the probability that you made the wrong decision?
4. 10 people toss their hats and each person randomly picks one. The experiment is repeated one more time.
 - (a) What is the probability that Bob picked his own hat both times?
 - (b) Let A be the event that at least one person picked their own hat both times. True or false: $P(A) > 25\%$? Justify your answer.
5. You are offered to play the following game for \$2: Toss three 3-sided dice and collect D dollars, where D is the number of distinct face values that appear.
 - (a) Is it rational for you to play this game? Justify your answer.
 - (b) Apply the Central Limit Theorem to estimate the probability that you come out ahead (you earn some money) after playing 90 rounds of the game.