

Questions

1. A Chicken lays a Poisson(λ) number N of eggs. Each egg independently hatches a chick with probability p . Let X be the number of chicks that hatch. Calculate
 - (a) the conditional expectation $E[X|N = n]$;
 - (b) the unconditional expectation $E[X]$;
 - (c) the unconditional expectation $E[NX]$;
 - (d) the covariance $\text{Cov}[X, N]$.
2. You draw 10 balls at random among 15 red and 5 blue balls. Let X be the number of red balls drawn.
 - (a) What is the expected value of X ?
 - (b) Write $X = X_1 + X_2 + \cdots + X_{10}$, where X_i indicates if the i -th drawn ball is red. What is the variance of X_i ?
 - (c) What is the covariance of X_i and X_j ($i \neq j$)?
 - (d) What is the variance of X ?
3. Two typing monkeys sit at special keyboards. The keyboards have only two letters, a and b. Each monkey types in a random 200 letter string, independently of the other one. Let E be the event "There is a pattern of 20 consecutive letters that appears in both strings." Show that $P(E) < 5\%$.
4. 100 people put their hats in a box and each one pulls a random hat out.
 - (a) Let G be any 10-person group. What is the probability that everyone in G pulls their own hat?
 - (b) What is the expected *number* of 10-person groups in which everyone pulls their own hat?
 - (c) Show that the probability that 10 or more people pull their own hat is less than 10^{-6} .

Additional ESTR 2018 question

5. In ESTR 2018 Lecture 9 I claimed that $E[X^4] = 3\sigma^4$ for a Normal($0, \sigma$) random variable X . In this exercise you will derive that formula.
 - (a) Let X be a Normal($0, 1$) random variable. Show that $E[e^{tX}] = e^{-t^2/2}$ for every real number t .
 - (b) Calculate $E[X^4]$ by taking derivatives. (You may assume that the expectation of a derivative is the same as the derivative of an expectation.)
 - (c) Can you calculate $E[X^k]$ for all k ?