

Practice questions

- Let X and Y be independent random variables with PMFs $P(X = 1) = P(X = 2) = P(X = 3) = 1/3$ and $P(Y = 3) = P(Y = 4) = 1/2$. Let $M = X + Y$ and $N = Y - X$.
 - What is the PMF of M given N ?
 - Are M and N independent? Justify your answer.
 - What is the expectation of M given $N < 2$?
- A fair coin is tossed 9 times. Which of the following random variables are independent?
 - The number of consecutive heads (HH) in the first 5 tosses and the number of consecutive tails (TT) in the last 5 tosses.
 - The number of changes in the first 5 tosses and the number of changes in the last 5 tosses. A *change* is a head followed by a tail or a tail followed by a head (TH or HT).
 - (Optional)** The number of tails followed by a head (TH) in the first 5 tosses and the number of heads followed by a tail (HT) in the last 5 tosses.
- You go to the casino with \$3 to play roulette. (Roulette has 37 possible outcomes, out of which 18 are red, 18 are black, and one is green.) Calculate the expected value and standard deviation of your profit under the following two gambling strategies:
 - You play for 3 rounds, where in every round you bet \$1 on red.
 - You bet all your money on red. If you win, you bet everything on red again. If you win again, you bet everything on red one last time.
- Find the expectation of the following random variables without calculating their PMF:
 - The first time X at which both the patterns TH and HT have appeared in a sequence of fair coin flips.
 - The first time Y at which all three face values have appeared in a sequence of rolls of a fair 3-sided die.

Additional ESTR 2018 questions

- Let T be the number of times a 6-sided die is rolled until a 6 appears. What is the expected value of T conditioned on all rolls producing even numbers? (*due to E. Mossel*)
- An election between two candidates T and B is coming up. Your job is to poll n random subjects about which candidate they will vote for. Some of your subjects, however, refuse to disclose their vote. Assume each subject supports exactly one of the two candidates and the ones who choose to disclose their vote always answer truthfully.
 - Let r_T and r_B be the fraction of T 's and B 's respective supporters who refuse to disclose their vote. Assuming you know the values r_T and r_B , show how to calculate an estimate X whose expected value is the fraction of T 's supporters. What is the variance of X ?
 - Now assume you know the value of r_T but not the value of r_B . What should you do?
 - What if you know neither r_T nor r_B ?