A Game Theoretic Analysis on Incentive Mechanisms for Wireless Ad Hoc VoD Systems

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Abstract—**Wireless ad hoc networks enable the wireless devices to directly communicate with each other. An emerging application in such systems is video-on-demand service that can greatly reduces the content server's workload by utilizing the nodes' resources. Such an application relies on the cooperation of all participating nodes. However, the nodes are selfish in nature. Therefore, a key design issue is to appropriately incentivize the cooperation among each other. In this paper, we design a simple yet effective reward based incentive mechanism so as to stimulate the nodes to upload and/or forward data to one another. By using a Stackelberg game model, we analyze the interactions between the content provider's rewarding strategy and the nodes' contributing behaviors. We derive a unique Stackelberg equilibrium and show its efficiency. By using a repeated game, we study the long term intelligent interaction between nodes and the content provider. We also design a cheating prevention mechanism and analyze its effectiveness under the repeated game setting.**

I. Introduction

Wireless ad hoc networks have become popular in the recent years. Such systems are unstructured in nature wherein the wireless devices can directly connect with each other and transmit data packets. Two nodes can mutually upload data to each other if they are geographically located within the transmission range, or they can rely on the cooperative intermediate nodes so that data packets can be sent to the destination node via multihop transmissions.

With the increasing popularity of mobile devices and the ease of their configuration, wireless ad hoc networks can find numerous applications, e.g., military networks, sensor networks, and emergency communications during natural disasters. In this paper, we focus on another emerging application in such systems, i.e., the Video-on-Demand (VoD) service, wherein nodes share video data among each other to satisfy the viewing requirement. Content servers may still exist in such systems, and the collaboration among nodes greatly reduces the workload of the content server. Such services can find its applications in educational institutes where students can share lecture videos on their phones or tablet computers, or in museums where visitors get self-guided services by watching the videos on demand using mobile electronic devices [9].

Note that data uploading and forwarding operations in a wireless ad hoc VoD system incur a cost at the nodes (e.g., battery energy consumption). Due to the selfish nature of nodes, free riding can happen in such systems wherein nodes do not have the incentive to contribute resource for other nodes. Thus, designing an effective and practical incentive scheme becomes critical to encourage nodes to contribute to the system, and thereby improving the system performance.

Authors in [2], [5], [12], [14] proposed various incentive mechanism designs for general ad hoc architecture, however, very limited work has addressed the VoD application in such networks. The existing incentive schemes for wireline VoD system cannot be directly applied to wireless ad hoc networks. Designing such a mechanism is challenging due to the following reasons. First, nodes in a wireless network are randomly geographically distributed, so that the system designer needs to make intelligent decisions on how to utilize the system's resources (e.g., using ad hoc transmission via nodes or letting the content server directly delivers data to requesting nodes). Second, VoD service needs to guarantee each node receives the video at the playback rate, and this requires a high transmission rate. Last but not least, it is difficult to monitor the nodes' behaviors in a wireless environment, so it is important to have guarantee that nodes honestly contribute their local resources. In this paper we present a simple yet effective incentive mechanism for wireless ad hoc VoD systems. The incentive is based on reward according to the data transmission rate that a node contributes. Our contributions are:

- We propose an incentive scheme for nodes to contribute their transmission rate. We model the interaction between the content provider and the nodes by using a *Stackelberg game*, and analyze the stability and efficiency of our incentive scheme.
- We use a repeated game to analyze their interactions and effectiveness of nodes' threatening strategy.
- We design a cheat prevention mechanism and use repeated game model to validate its effectiveness.

Our paper is organized as follows. Section II and III introduce the system model and the game theoretic model, respectively. Then we analyze the Stackelberg game in Section IV, and use a repeated game model to extend our results for cheating prevention design and threatening analysis in Section V. Section VI states related work and Section VII concludes.

II. System Model

We consider a wireless ad hoc Video-on-Demand system which consists of nodes and content servers. Peers in the system seek to download the video data at the playback rate r. At the same time, they need to (1) upload their cached data to other nodes, and/or (2) receive data from one node and forward them to another node. Also, a node can download the video data from (1) a nearby node within the transmission range which caches the data (e.g., nodes that are within the one hop transmission), or (2) a remote node out of the transmission range which sends the data via intermediate nodes' forwarding operations. In this case, multiple hops transmission will occur. Due to the mobility nature of nodes, a node may not be able to receive the video data at the playback rate from the above two sources. In such a case, a content server needs upload the remaining data so as to guarantee the quality of service. The content servers are geographically distributed in the whole service area; however, in our model, we consider a single logical server which provides the download service.

Contributing the local resources will incur a cost to the energy, therefore, the selfish nodes are not willing to contribute by default. Hence, they need to be incentivized to contribute their (a) data transmission rate to upload and/or forward data; and (2) storage space to cache some data. Both aspects are equally important since a node will fail to contribute if it either does not provide the transmission rate, or does not cache the data. In this paper, we focus on incentivizing the data transmission rate (i.e., upload and download rate) with an ideal assumption that the system has implemented an incentive mechanism for caching so that the video data are properly cached by the nodes.

We design an incentive mechanism under which the content provider rewards the nodes based on the data transmission rate they contribute. The data transmission rate includes the upload rate to other nodes, as well as the download rate for data forwarding. Once a node dedicates its transmission rate, it incurs an energy cost, while the content provider can benefit from reducing the high workload at the content server and hence reducing the operational cost. Therefore, the content provider *rewards* the nodes to contribute. The reward can be in various forms, e.g., real money rebate for the service fee and virtual credits or reputation record for advanced services. Note that any reward scheme can be represented by the currency flow from the content provider to the nodes. Even for rewards in virtual currency or reputation, they imply that the P2P-VoD operator needs to invest money for developing advanced/prioritized services for users. We do not restrict the form of implementing the rewards in our paper; however, we use an abstract model to describe the reward in terms of monetary value.

We define the reward W to a node to be a function of its dedicated data transmission rate u. In general, this function can be in any form, in this presentation, we restrict it to the linear case, i.e., $W(u) = wu$, where w is the unit reward. The linear reward scheme can be easily understood by the nodes and implemented by the content provider in practice.

III. **Stackelberg Game Model**

In this section, we present the Stackelberg game model to analyze the interactions between the content provider and the nodes. The content provider needs to decide the per rate reward w to the nodes, while nodes need to decide the amount of data transmission rate u. For simplicity of presentation, we assume that nodes are homogeneous and use the same strategy u in the game, and the model can be easily extended to the heterogeneous case. In what follows, the game is between the content provider and any particular node. Due to the homogeneous assumption, it may seem like a game between the content provider and a set of nodes, while in reality these nodes are independent among themselves.

The content provider aims at minimizing its total cost, i.e., the cost of uploading and the cost of rewarding the nodes. Assume the upload cost is a linear function in the upload rate of the server, then we define the utility of the content provider as the following:

$$\pi_s(w,u) = -c_s\left(Nr - N\int_0^u v(x)dx\right) - Nwu, \quad (1)$$

where N denotes the total number of nodes, r denotes the playback rate of a video, and $v(x)$ denotes the marginal reduction of upload rate at the server with respect to any particular node's dedicated transmission rate x. Note that we implicitly assume the system can fully utilize all nodes' rate contributions. For simplicity of notations, we define

$$C_s(u) = c_s\left(Nr - N\int_0^u v(x)dx\right) \quad (2)$$

and therefore,

$$\pi_s(w,u) = -C_s(u) - Nwu. \quad (3)$$

Similarly, we define the utility of a node as the reward it receives, minus its cost for data uploading and forwarding:

$$\pi_p(u,w) = wu - C_p(u), \quad (4)$$

where $C_p(u)$ denotes the cost of dedicating u amount of transmission rate.

To maximize their utilities, the content provider solves the optimization problem $\max_w \pi_s(w,u)$, and the nodes solves $\max_u \pi_p(u,w)$. We consider a Stackelberg game [8] where the content provider decides w first, and after that, the nodes decide u. It is natural to assume the content provider as the first-mover whereas the nodes response to the reward w accordingly. Once u is determined, the content provider will keep its reward scheme stable so as to manage a creditable service. To obtain the Stackelberg equilibrium of the game, we use the backward induction technique [8]. In particular, each node solves the problem $u^*(w) = \mathrm{argmax}_u\, \pi_p(u,w)$ given any w. By knowing the nodes' best responses, the content provider solves the problem $w^* = \mathrm{argmax}_w\, \pi_s(w, u^*(w))$.

In the following analysis, we assume $C_p(u)$ satisfies the following property:

- $C_p(u)$ is continuous and twice differentiable in u.

- $C_p(0) = 0$, $C_p'(u) > 0$, $C_p''(u) \geq 0$.

$C_p'(u) > 0$ means that a node's cost increases with its dedicated transmission rate. While $C_p''(u) > 0$ means the marginal cost also increases with the dedicated rate. The above assumptions reflect the fact that a node's viewing performance would not be affected too much if it contributes a small amount of transmission rate; however, if a node dedicates a large amount of transmission rate, its battery may drain quickly, and the performance of video might be substantially reduced as much of the bandwidth is used for uploading and forwarding data to other nodes.

IV. Analysis of Stackelberg Equilibrium

In this section, we will discuss the properties of our Stackelberg equilibrium, in particular, its existence, uniqueness and efficiency. We start by a simplified scenario to explore the feature of $v(u)$.

A. Simplified scenario

We first assume there is no transmission or congestion loss. Data transmission can always reach the destination as long as it is within the transmission range of the sender. Assume the server is aware of the geographical distribution of nodes and use deterministic routing, i.e., a routing flow $(P_s, P_1, ..., P_n, P_d)$ is determined before transmission starts where P_s is the source and P_d is the destination. Note that a request may need multiple hops transmission since the source and destination can be far from each other. Obviously, one hop transmission is the most efficient utilization of node's contribution, where one unit transmission volume requires one unit volume from node's contribution. However, if the data packet is transmitted via k hops, then the $k-1$ intermediate nodes need to receive the packet and forward it, which causes $2(k-1)$ unit volume of nodes' transmission rate contribution. Adding the upload contribution of the source node, one unit of a k hops transmission requires $2k-1$ units of nodes' contribution in total. Let U_k be the total rate of data packets whose required transmission hops is larger than or equal to k. In this paper, we do not address the data scheduling policy, but assume U_k is known and that the system utilize the nodes' rate resources to satisfy smaller hops transmission first (since the utilization is more efficient). Then the marginal reduction of server's upload rate at the node's transmission rate u is

$$v(u) = \frac{1}{2k-1} \quad \text{if } U_{k-1} \leq u < U_k. \tag{5}$$

Using backward induction for the Stackelberg game, we can have the following claim.

Theorem *1:* Assume $C_p''(u) = 0$. At the Stackelberg Equilibrium, we have $u^* = \text{argmax}_{U_k}\{C_p'(U_k) \leq \frac{c_s}{2k-1}\}$, and $w^* = C_p'(u^*)$.

Proof: Denote by (u^*, w^*) as the nodes' transmission rate and the content provider's unit reward at the Stackelberg equilibrium, respectively. Then from a node's perspective, for any given w, it chooses $u^*(w)$ that maximizes its utility, i.e.,

$$r^*(w) = \underset{u}{\text{argmax}}\, \pi_p(u, w) = \underset{u}{\text{argmax}}[wu - c_p(u)]. \tag{6}$$

Due to the continuity assumption on $C_p(u)$, we have

$$\left.\frac{\partial \pi_p(u, w)}{\partial u}\right|_{u^*} = w - C_p'(u^*). \tag{7}$$

By letting the above formula equal zero, we have

$$C_p(u^*) = w. \tag{8}$$

From the content provider's perspective, it aims at maximizing its own utility. Given Eq. 8, we can write down the utility of the content provider in terms of u^*:

$$\begin{aligned}\pi_s(w, u^*(w)) &= \pi_s(u^*) \\ &= -c_s\left(Nr - N\int_0^{u^*} v(x)dx\right) - NC_p(u^*)u^*.\end{aligned} \tag{9}$$

Therefore,

$$\frac{\partial \pi_s(u^*)}{\partial u^*} = N[c_s v(u^*) - C_p'(u^*) - C_p''(u^*)u^*]. \tag{10}$$

Let $C_p''(u) = 0$. It is easy to verity that $\frac{\partial \pi_s(u^*)}{\partial u^*}$ is deceasing in u^*. Hence, the content provider will set w such that the corresponding $u^*(w)$ is the maximal value to guarantee $\frac{\partial \pi_s(u^*)}{\partial u^*} \geq 0$. Noting Eq. 5, we have $u^* = \text{argmax}_{U_k}\{C_p'(U_k) \leq \frac{c_s}{2k-1}\}$, and $w^* = C_p'(u^*)$. ■

Denote k^* such that $u^* = U_{k^*}$. It is obvious to see that at the Stackelberg equilibrium, the content provider will direct all data requests that can be transmitted within k^* hops to other nodes using ad hoc transmission, while the server will upload to the nodes which request data k hops away.

B. General case analysis

The analysis in the previous subsection provides a neat form of the Stackelberg equilibrium. Nevertheless, in a realist wireless ad hoc system, transmission loss cannot be neglected, and the system may implement sophisticated routing policies (e.g., optimistic routing [1]). Hence, Eq. (5) may not be realistic. However, we can still assume similar features on $v(u)$. In particular, we assume $v(u)$ is a non-increasing continuous function in u. For technical simplicity, we also assume the nodes' and content provider's decision are upper bounded by $\overline{u}$ and $\overline{w}$, respectively. In what follows, we relate the Stackelberg equilibrium and an optimization problem, and analyze the existence, uniqueness and efficiency of the Stackelberg equilibrium.

Optimization framework: We first state the following lemma, which establishes the relationship between the Stackelberg equilibrium with an optimization problem:

Lemma *1:* If u^* is a solution to the following problem:

$$\min_u C_s(u) + NuC_p'(u), \tag{11}$$

then there exists a Stackelberg equilibrium $(u^*, u^*C_p'(u^*))$; further, if (u^*, w^*) is a Stackelberg equilibrium, then u^* is the solution to problem (11).

Proof: We start by showing the first half of the statement. Denote $u^* = \text{argmin}_u[C_s(u) + NuC_p'(u)]$ and $w^* = u^*C_p'(u^*)$.

We show that (u^*, w^*) is a Stackelberg equilibrium. Since $\pi_p(u,w)$ is strictly concave in u, so for any given w^*, if u^* satisfies $u^* C_p'(u^*) = w^*$, then u^* maximizes the nodes' utility $\pi_p(u, w^*)$. Hence, the nodes do not have incentives to deviate from u^*. Suppose the content provider has an incentive to deviate from w^* and can obtain higher utility by setting $w = w_0$, where the nodes' response is to set $u = u_0$ so that u_0 maximizes $\pi_p(u, w_0)$. Because of the strict concavity of $\pi_p(u,w)$, there are only three possible cases:

(1) $C_p'(u_0) = w_0$ if $C_p'(0) \le w_0 \le C_p'(\bar{u})$; or
(2) $u_0 = 0$ if $C_p'(0) > w_0$; or
(3) $u_0 = \bar{u}$ if $C_p'(\bar{u}) < w_0$.

For any of the above cases, we have

$$\begin{aligned} & C_s(u_0) + N u_0 C_p'(u_0) \le C_s(u_0) + N u_0 w_0 \\ & < C_s(u^*) + N u^* w^* = C_s(u^*) + N u^* C_p'(u^*). \end{aligned} \tag{12}$$

The first inequality holds for the above three cases. The second inequality holds because we assume the content provider can have higher utility by setting $u = u_0$ instead of $u = u^*$. However, $C_s(u_0) + N u_0 C_p'(u_0) < C_s(u^*) + N u^* C_p'(u^*)$ contradicts the fact that u^* is a solution of (11). This implies that the content provider has no incentive to deviate from w^*. Given that we have shown the nodes do not have any incentive to deviate from u^* given any w^*, we conclude (u^*, w^*) is a Stackelberg equilibrium[1].

To show the second half of the statement, suppose there exists a Stackelberg equilibrium (u^*, w^*), but u^* is not a solution to (11), i.e., there exists $u_0 \ne u^*$ such that $C_s(u_0) + N u C_p'(u_0) < C_s(u^*) + N u C_p'(u^*)$. Assume the content provider sets $w_0 = u_0 C_p'(u_0)$. Taking the derivative in (4) and noting the strict concavity of $\pi_p(u,w)$, we have the nodes' unique best response is $u = u_0$ for given $w_0 = u_0 C_p'(u_0)$. Therefore, $\pi_s(w_0, u_0) = -C_s(u_0) - N u_0 C_p'(u_0) > -C_s(u^*) - N u^* C_p'(u^*) = \pi_s(w^*, u^*)$, which contradicts to the fact that (u^*, w^*) is a Stackelberg equilibrium. This implies u^* must be a solution to (11). ∎

Existence and uniqueness: The following theorem states the existence and condition for the uniqueness of the Stackelberg equilibrium.

Theorem *2:* The Stackelberg equilibrium always exists. If $u C_p'(u)$ is strictly convex in u, then the nodes' solution u^* at the Stackelberg equilibrium is unique.

Proof: We first show the existence. The nodes solve $\max_u \pi_p(u,w) = wu - C_p(u)$. For any given w, π_p is continuous and strictly concave in u over the compact set $[0, \bar{u}]$. Hence, the optimal solution $u^*(w) = \text{argmax}_u \pi_p(u,w)$ exists and is unique. Substituting u by $u^*(w)$ in $\pi_s(w,u)$, the provider's utility $\pi_s(w, u^*(w))$ is continuous in w over the compact set $[0, \bar{w}]$, so $w^* = \text{argmax}_w \pi_s(w, u^*(w))$ exists.

Next we show the uniqueness of u^* when $u C_p'(u)$ is strictly convex in u. Since $v(u)$ is non-increasing, $C_s(u)$ is convex in u, and $u C_p'(u)$ is strictly convex in u, we can observe that the problem (11) is a strictly convex minimization over a compact set, which has a unique solution. According to Lemma 1, any Stackelberg equilibrium (u^*, w^*) satisfies that u^* is a solution to (11). Therefore, we conclude that the nodes' solution in the Stackelberg equilibrium is unique[2]. ∎

Efficiency: For simplicity, in the rest of this subsection, we assume $C_s(u)$ and $C_p(u)$ are both twice differentiable in u.

We define the social welfare, π_w, as the sum of the content provider's and all nodes' utilities:

$$\pi_w(u) = \pi_s + N\pi_p = -c_s\left(Nr - N\int_0^u v(x)dx\right) - NC_p(u). \tag{13}$$

Define $u_w = \text{argmax}_u \pi_w(u)$, and u^* as the nodes' solution at the Stackelberg equilibrium. We have the following result.

Lemma *2:* The nodes' transmission rate at the Stackelberg equilibrium is no larger than the value that maximizes the social welfare, i.e., $u^* \le u_w$.

Proof: Denote $C_{SW}(u) = -\pi_w(u) = C_s(u) + NC_p(u)$, and $C_{SE}(u) = C_s(u) + u C_p'(u)$. Maximizing the social welfare is equivalent to solving $\min_u C_{SW}(u)$, $u \in [0, \bar{u}]$. According to Lemma 1, u^* can be obtained by solving $\min_u C_{SE}(u)$, $u \in [0, \bar{u}]$. Therefore, u_w and u^* are the minimizers to $C_{SW}(u)$ and $C_{SE}(u)$, respectively. By taking the first order derivative, we have

$$C_{SW}'(u) = C_s'(u) + NC_p'(u), \tag{14}$$

$$C_{SE}'(u) = C_s'(u) + NC_p'(u) + NuC_p''(u). \tag{15}$$

There are only two possible cases regarding $C_{SW}'(u)$:

(1) If $C_{SW}'(u) > 0$, $\forall u \in [0, \infty)$, then $u_w = 0$. Since $NuC_p''(u) \ge 0$, we have $C_{SE}'(u) = C_{SW}'(u) + NuC_p''(u) > 0$, $\forall u \in [0, \infty)$, so $u^* = 0 = u_w$.

(2) If there exists a $u_{SW} \in [0, \infty)$ such that $C_{SW}'(u_{SW}) = 0$, then u_{SW} must be unique due to the strict convexity of $C_{SW}(u)$. We have $u_w = \max(u_{SW}, \bar{u})$. By the concavity assumption on $C_p(u)$ and non-increasing assumption on $v(u)$, $C_s'(u)$ and $NC_p'(u)$ are both non-decreasing in u and $NuC_p''(u) > 0$. Hence, for any $u > u_{SW}$, we have $C_{SE}'(u) > C_{SW}'(u) > C_{SW}'(u_{SW}) = 0$. This implies any $u > u_w = \max(u_{SW}, \bar{u})$ cannot be the minimizer of $C_{SE}(u)$, $u \in [0, \bar{u}]$. Therefore, $u^* \le u_w$.

Combining the results in the above two cases, we have $u^* \le u_w$. ∎

Define the *rate utilization* (RU) as the ratio of the transmission rate at the Stackelberg equilibrium, to the value which

[1]Noting the above three cases and that the content provider aims at maximizing its utility, if $u^* > 0$, then the corresponding Stackelberg equilibrium is unique where $w^* = u^* C_p'(u^*)$. If $u^* = 0$, then any (u^*, w^*) where $0 \le w^* \le C_p'(0)$ is a Stackelberg equilibrium.

[2]We do not claim the Stackelberg equilibrium is unique. The only chance of having multiple Stackelberg equilibria is $u^* = 0$, where any (u^*, w^*) with $0 \le w^* \le C_p'(0)$ is a Stackelberg equilibrium. When $u^* > 0$, the Stackelberg equilibrium is unique, where the content provider sets $w^* = u^* C_p'(u^*)$.

maximizes the social welfare, i.e.,

$$RU = \frac{u^*}{u_w}. \tag{16}$$

According to the above lemma, we know that in general, $RU \leq 1$, and when RU approaches 1, it indicates that the system is in an efficient state. In particular, we have the following theorem.

Theorem *3:* If $u^* > 0$ and $C_p''(u)$ is increasing in u, then $RU \geq \frac{1}{2}$.

Proof: Since $u_w \geq u^* > 0$, there must exist unique u_{SW} and u_{SE} such that $C_{SW}'(u_{SW}) = 0$, $C_{SE}'(u_{SE}) = 0$. Thus, we have $u_w = \max(u_{SW}, \bar{u})$ and $u^* = \max(u_{SE}, \bar{u})$.

Since $u_w \geq u^*$, we have $BU = \frac{u^*}{u_w} \geq \frac{u_{SE}}{u_{SW}}$. Noting the form of $C_{SW}'(u)$ and $C_{SE}'(u)$, we have

$$\begin{aligned} C_s'(u_{SW}) + NC_p'(u_{SW}) &= 0, & (17)\\ C_s'(u_{SE}) + NC_p'(u_{SE}) + Nu_{SE}C_p''(u_{SE}) &= 0. & (18) \end{aligned}$$

Since $C_s'(u)$ and $C_p'(u)$ are both continuous functions in u, there must exist $u_1, u_2 \in [u_{SE}, u_{SW}]$ such that

$$\begin{aligned} &C_s''(u_1)(u_{SW} - u_{SE}) + \\ &\quad NC_p''(u_2)(u_{SW} - u_{SE}) - Nu_{SE}C_p''(u_{SE}) = 0. \end{aligned} \tag{19}$$

Hence, we have

$$\frac{u_{SW} - u_{SE}}{u_{SE}} = \frac{NC_p''(u_{SE})}{C_s''(u_1) + NC_p''(u_2)} \leq \frac{C_p''(u_{SE})}{C_p''(u_2)}. \tag{20}$$

Since $C_p(u)$ is increasing in u and $u_{SE} \leq u_2$, we have

$$\frac{u_{SW}}{u_{SE}} \leq 1 + \frac{C_p''(u_{SE})}{C_p''(u_2)} \leq 2, \tag{21}$$

and thus

$$BU = \frac{u^*}{u_w} \geq \frac{u_{SE}}{u_{SW}} \geq \frac{1}{2}. \tag{22}$$

■

The above theorem requires that $C_p''(u)$ increases in u. If this condition does not satisfy, it is in general difficult to characterize RU. In the following theorem, we choose a special form of the nodes' cost function and derive a corresponding efficiency bound.

Theorem *4:* If $u^* > 0$ and $C_p(u) = c_p u^\beta (1 \leq \beta \leq 2)$, then $RU > \frac{\beta - 1}{\beta}$.

Proof: Using the similar approach in the previous proof, we have

$$\frac{u_{SW}}{u_{SE}} \leq 1 + \frac{C_p''(u_{SE})}{C_p''(u_2)} \leq 1 + \frac{C_p''(u_{SE})}{C_p''(u_{SW})}, \tag{23}$$

where $u_2 \in [u_{SE}, u_{SW}]$. The second "$\leq$" holds because $C_p''(u)$ is a decreasing function in u and $u_2 \leq u_{SW}$.

Noting the form of $C_p(u)$, we have

$$\frac{u_{SW}}{u_{SE}} \leq 1 + \frac{u_{SE}^{\beta-2}}{u_{SW}^{\beta-2}} = 1 + \left(\frac{u_{SW}}{u_{SE}}\right)^{2-\beta}. \tag{24}$$

Define $x = \frac{u_{SW}}{u_{SE}}$ and $f(x) = x - x^{2-\beta}$. Let $g(x) = f(x) - (\beta - 1)x + \beta - 1$. Then it is easy to verify that $g'(x) > 0, \forall x \geq 1$ and $g(0) = \beta - 1 > 0$. Hence, $f(x) > (\beta - 1)x - (\beta - 1), \forall x \geq 1$.

Hence, in order to satisfy $f(x) \leq 1$, we must have

$$(\beta - 1)x - (\beta - 1) < 1, \tag{25}$$

which indicates $x = \frac{u_{SW}}{u_{SE}} < \frac{\beta}{\beta - 1}$. Therefore,

$$BU = \frac{u^*}{u_w} \geq \frac{u_{SE}}{u_{SW}} > \frac{\beta - 1}{\beta}. \tag{26}$$

■

Numerical examples: We use numerical examples to verify the efficiency of the Stackelberg equilibrium. We have the following settings:

- $C_s(u) = N(r - u^\gamma)$; $C_p(u) = c_p u^\beta$;
- $N = 10,000$, $r = 500$kbps, $\gamma = 0.8$, $u \in [0, 1000]$kbps.

In Fig. 1(a), we set $\beta = 2.2$ (corresponding to Theorem 3), vary $c_p \in [0.00006, 0.00028]$ and plot the corresponding u^* and u_w. We can observe that u^* is always smaller than u_w. In Fig. 1(b), we compare the social welfare at the Stackelberg equilibrium vs. the maximal social welfare, which are close to each other in general. In Fig. 2(a) and Fig. 2(b), we set $\beta = 1.2$ (corresponding to Theorem 4) and vary $c_p \in [0.06, 0.28]$. We can observe very similar results as the previous example. These numerical results validate the efficiency of the Stackelberg equilibrium.

V. Extensions under Repeated Game Model

A. Peer threatening

In the previous section, we use one-shot interaction of content provider and the nodes using a Stackelberg game. In general, this interaction can last a long time. We use a *repeated game model* to discuss whether the nodes can perform "smarter" in this case.

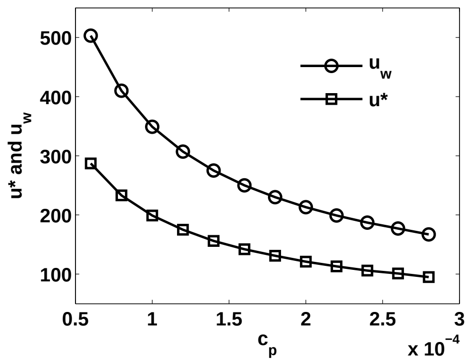
Assume the game is played infinitely long. At round t, the utilities of the content provider and the nodes are denoted by $\pi_s(t)$ and $\pi_p(t)$ respectively. Their utilities in the repeated game are

$$\Pi_s = \sum_{t=1}^{\infty} \delta_s^t \pi_s(t), \qquad \Pi_p = \sum_{t=1}^{\infty} \delta_p^t \pi_p(t); \tag{27}$$

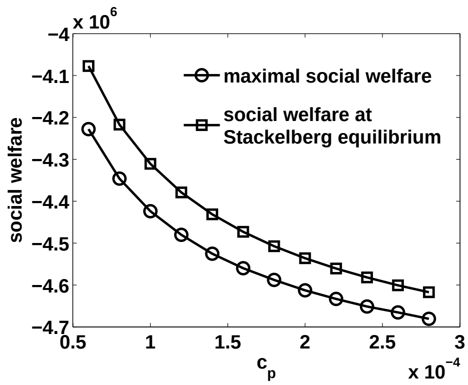
where δ_s and δ_p denote their discount factors.

An interesting difference of the repeated game from the one-shot game is that the nodes may have incentives to deviate from the Stackelberg equilibrium: the nodes may threaten to punish the content provider (e.g., contributing zero transmission rate) unless the content provider sets the reward higher than that in Stackelberg equilibrium.

Assume the nodes request the content provider to set $w = \tilde{w}$, and threaten that if the content provider refuses to do so, they will set $u = 0$. We first consider what are the possible interactions in a particular round. Responding to this threat, the content provider has two possible strategies: (1) it compromises and sets $w = \tilde{w}$; or (2) it resists the threat and

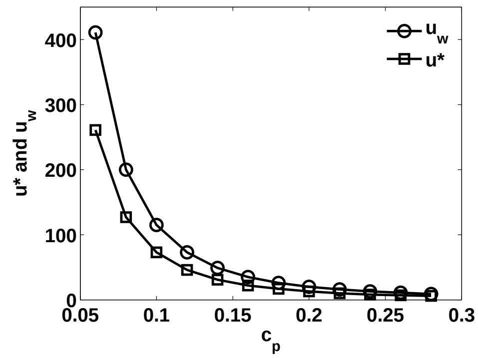


(a) Comparing u^* and u_e

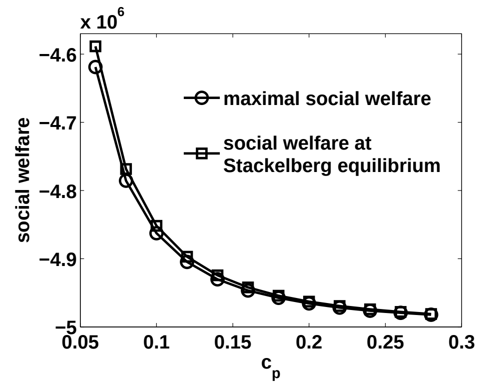


(b) Comparing social welfare

Fig. 1. Efficiency of Stackelberg Equilibrium ($\beta = 2.2$)



(a) Comparing u^* and u_e



(b) Comparing social welfare

Fig. 2. Efficiency of Stackelberg Equilibrium ($\beta = 1.2$)

sets $w = w^*$ where w^* is the Stackelberg equilibrium. We denote by $\mathcal{C}$ and $\mathcal{R}$ the compromising and resisting strategy of the content provider. If the content provider plays $\mathcal{R}$, then the nodes have two possible choices: (1) they punish the content provider, i.e., setting $u = 0$; or (2) they do not carry out the threat at all; rather, they accept w^* and set $u^* = \mathrm{argmax}_u\, \pi_p(u, w^*)$. We denote by $\mathcal{P}$ and $\mathcal{A}$ the punishing and accepting strategy of the nodes. If the content provider plays $\mathcal{C}$, then the nodes will surely accept this reward and set $\tilde{u} = \mathrm{argmax}_u\, \pi_p(u, \tilde{w})$. We still use $\mathcal{A}$ to denote this accepting strategy for $u = \tilde{u}$. Therefore, there are three cases of possible interactions in a particular round: $(\mathcal{R}, \mathcal{P})$, $(\mathcal{R}, \mathcal{A})$ and $(\mathcal{C}, \mathcal{A})$, where the two elements in each pair denote the content provider's and the nodes' strategies, respectively. We use $\hat{\pi}_s$, π_s^* and $\tilde{\pi}_s$ (resp. $\hat{\pi}_p$, π_p^* and $\tilde{\pi}_p$) to denote the one-shot utility of the content provider (resp. the nodes) for the above three cases respectively. In general, we have $\pi_s^* > \tilde{\pi}_s > \hat{\pi}_s$ and $\tilde{\pi}_p > \pi_p^* > \hat{\pi}_p = 0$.

We define a threat to be *credible* if, under the triggering condition, the threat-claimer would obtain no less utility by indeed carrying out the threat than not carrying out the threat. In particular, the nodes' threat is credible if the nodes can achieve higher (or at least equal) utility by playing $\mathcal{P}$ compared to $\mathcal{A}$ if the content provider plays $\mathcal{R}$ in an arbitrary round. Now we discuss under what condition the nodes' threat is credible.

Theorem *5:* If $\lfloor \log_{\delta_s} \frac{\tilde{\pi}_s - \hat{\pi}_s}{\pi_s^* - \hat{\pi}_s} \rfloor \leq \lfloor \log_{\delta_p} \frac{\pi_p^* - \tilde{\pi}_p}{\tilde{\pi}_p - \hat{\pi}_p} \rfloor$, then the nodes' threat is credible; otherwise, it is not credible.

Proof: Suppose the content provider and the nodes have played $(\mathcal{C}, \mathcal{A})$ in the first $(t_0 - 1)$ rounds, and that the content provider plays $\mathcal{R}$ in round t_0. There are two possibilities regarding the nodes: (1) the nodes play $\mathcal{P}$, resulting that $\pi_s(t_0) = \hat{\pi}_s$ and $\pi_p(t_0) = \hat{\pi}_p$; or (2) the nodes play $\mathcal{A}$, resulting $\pi_s(t_0) = \pi_s^*$ and $\pi_p(t_0) = \pi_p^*$. Whether the nodes really punish the content provider depends on how many rounds they need to play $\mathcal{P}$ before the content provider plays $\mathcal{C}$. In particular, denote by t_p the number of rounds the node can afford playing $\mathcal{P}$. After that, the content provider changes back to $\mathcal{C}$. We have:

$$\sum_{t=1}^{t_0-1} \delta_p^t \tilde{\pi}_p + \sum_{t=t_0}^{t_0+t_p-1} \delta_p^t \hat{\pi}_p + \sum_{t=t_0+t_p}^{\infty} \delta_p^t \tilde{\pi}_p \geq \sum_{t=1}^{t_0-1} \delta_p^t \tilde{\pi}_p + \sum_{t=t_0}^{\infty} \delta_p^t \pi_p^*. \tag{28}$$

The left side term represents a particular node's total utility if the nodes play t_p rounds of punishing strategy $\mathcal{P}$ when the content provider plays resisting strategy $\mathcal{R}$, and after that, the content provider compromises to set back $w = \tilde{w}$; the right side term represents a particular node's total utility if the nodes accept $w = w^*$ and set $u = u^*$ from t_0 onwards. If (28) holds, it is beneficial for the nodes to play $\mathcal{P}$ for t_p rounds; otherwise, the nodes are better off by playing $\mathcal{A}$ from t_0 onwards.

Similarly, denote by t_s the number of rounds that the content provider can afford playing $\mathcal{R}$ and being punished by the nodes. After that, the nodes play $\mathcal{A}$ from round t_0+t_s onwards and the content provider continues playing $\mathcal{R}$. We have:

$$\sum_{t=1}^{t_0-1}\delta_s^t\tilde{\pi}_s+\sum_{t=t_0}^{t_0+t_s-1}\delta_s^t\hat{\pi}_s+\sum_{t=t_0+t_s}^{\infty}\delta_s^t\pi_s^*\geq\sum_{t=1}^{\infty}\delta_s^t\tilde{\pi}_s. \quad (29)$$

If this condition holds, then the content provider has an incentive to play $\mathcal{R}$ from round t_0 onwards; otherwise, it is better for the content provider to play $\mathcal{C}$ in all time rounds.

The maximal value satisfying the above inequalities are $t_s=\lfloor\log_{\delta_s}\frac{\tilde{\pi}_s-\hat{\pi}_s}{\pi_s^*-\hat{\pi}_s}\rfloor$ and $t_p=\lfloor\log_{\delta_p}\frac{\pi_p^*-\hat{\pi}_p}{\tilde{\pi}_p-\hat{\pi}_p}\rfloor$. When $t_s>t_p$, it means the nodes will give up playing $\mathcal{P}$ before the content provider changes back to $\mathcal{C}$. In this case, the threat of the nodes is not credible: it is better for the nodes to accept $w=w^*$ when the content provider plays $\mathcal{R}$ in round t_0. Otherwise, i.e., when $t_s\leq t_p$, the nodes can play $\mathcal{P}$ for no larger than t_p rounds whereas the content server has to compromise and accept $w=\tilde{w}$, and therefore, the threat is credible. ∎

From Theorem 5, we can see that a larger gap between $\tilde{\pi}_p$ and π_p^*, or a larger discount factor δ_p, will induce a larger t_p, implying that the nodes' threat is more likely to be credible. Physically, it means if the potential benefit after threatening is large, or if the nodes care much about utility in future, the nodes will have higher incentives to punish the content provider in a few rounds so as to force it to reward more than in the Stackelberg equilibrium.

B. Cheating prevention guarantee

One important issue in ad hoc wireless networks is that transmissions have certain loss probability so that nodes may take this advantage to cheat. A node may declare that it has not received the data from its upstream node although it has already successfully received; or it may declare to have sent the data to downsreaming nodes while dropping the packets quietly. Once the data packets are lost, it is very difficult for the system to identify if any node dropped the packets intentionally.

We use the similar idea in [7]. For any node P_j, denote by P_{j-1} its upstream node and by P_{j+1} its downstream node. Define a unit time slot during which P_j receives d_j data packets from the upstream node. Define the following "*reporting policy*" $b_j(d_j)$ for node P_j: $b_j(d_j)=1$ if $d_j\geq m_j$ and $b_j(d_j)=0$ otherwise, where m_j is a pre-defined threshold. Physically, b_j represents whether a node receives enough number of packets as expected. If a node receives packets from its upstream node but cheats by dropping them intentionally, then $b_{j-1}\neq b_{j+1}$ and we regard it as cheating. If this node receives packets, honestly reports and forwards the data, then its upstream and downstream nodes are expected to have the same report b_{j-1} and b_{j+1}. However, note that the data loss can also be caused by transmission loss, hence we need to find suitable thresholds so as to prevent cheating while minimize the probability of misjudgment. Numerical computations in [7] show that one can decide such suitable thresholds. In this paper, we assume the following proposition satisfies.

Proposition *1:* Let $P(d_l|d_{l-1})$ be the conditional probability that P_l receives d_j packets given that its upsreaming node receives d_{l-1} packets. Then for any given $P(d_l|d_{l-1})$ for $j-1\leq l\leq j+1$, there exists m_{j-1}^*, m_j^* and m_{j+1}^* such that $P(b_{j-1}=b_j=b_{j+1})$ is maximized.

Let α be the maximal probability $P(b_{j-1}=b_j=b_{j+1})$ achieved in the above proposition. We develop a new repeated game framework to prohibit cheating. For every unit time slot, each node P_j reports b_j to the server. Once the server finds $b_{j-1}=b_j=b_{j+1}$ does not hold for any data flow through P_j, P_j is regarded to have cheated in the previous round (which might be misjudged with a small probability $1-\alpha$). If a node cheats, it can obtain a higher utility in this round; however, the content provider can receive the report soon and will punish P_j from the next time round and for T continuous rounds. We assume the nodes contributes no rate and receives no reward during the punishing rounds, leading to a utility of 0. After T rounds of punishment, this node still receives reward if it behaves normally. Denote π_0 as the one-shot utility of an honest node in a time round when $b_{j-1}=b_j=b_{j+1}$, and denote π_h as the one-shot utility that a cheating node can obtain in its cheating round. Let α be the discount factor in this repeated game. Intuitively, a node will not have an incentive to cheat if the punishment during the following rounds incurs a high utility deduction comparing with the extra utility achieved during the cheating round. Formally, we have

Theorem *6:* If $\frac{\alpha\delta^T-\delta^T+1-\alpha\delta^{T+1}}{\alpha\delta^T-\delta^T+1-\alpha\delta}\pi_0-\pi_h>0$ satisfies, then a node does not have an incentive to cheat.

Proof: We investigate whether a node has an incentive to cheat during time round t_0. Assume in round t_0, a node is regarded as honest. We first investigate its utility if it does not cheat in this round. Let $\Pi_0(t_0)$ be the expected total utility from t_0 until infinity if it does not cheat in round t_0. Note the one-shot utility of the node in this round is $\pi_0\delta^{t_0}$. In round t_0+1, this node is regarded as honest with probability α, and if this happens, the expected utility from t_0+1 until infinity is $\Pi_0(t_0)\delta$. This node may also be misjudged as cheating with probability $1-\alpha$. If this happens, this node receives zero utility in the following T rounds. The expected total utility from round t_0+T-1 until infinity is $(\Pi_0(t_0)-\pi_0\delta^{t_0})\delta^T$. Therefore, we have

$$\Pi_0(t_0)=\pi_0\delta^{t_0}+\alpha\Pi_0\delta+(1-\alpha)(\Pi_0(t_0)-\pi_0\delta^{t_0})\delta^T. \quad (30)$$

Hence,

$$\Pi_0(t_0)=\pi_0\delta^{t_0}\frac{1-\delta^T(1-\alpha)}{1-\alpha\delta-(1-\alpha)\delta^T}. \quad (31)$$

Next we investigate the expected total utility if the node cheats in round t_0. In that case, the node receives a high one-shot utility $\pi_h\delta^{t_0}$ in this round, but it is punished in the next T rounds. The expected utility from t_0 until infinity if the node cheats in round t_0 is

$$\Pi_h(t_0)=\pi_h\delta^{t_0}+(\Pi_0(t_0)-\pi_0\delta^{t_0})\delta^T. \quad (32)$$

If $\Pi_0(t_0)>\Pi_h(t_0)$, then the node does not have an incentive to cheat. Substituting Eq. 31 and Eq. 32 into the condition, we obtain the claim in the theorem. ∎

The above theorem points out the condition under which our cheating prevention mechanism is effective. Intuitively, when T is large, the punishment is significant so that nodes do not have incentives to cheat. A limiting case is when $\alpha = 1$, this condition satisfies if and only if

$$T > \log_\delta(1 - \frac{\pi_h}{\pi_0}(1-\delta)) - 1.$$

VI. Related Work

Ad hoc wireless networks have received lots of research interests. Many incentive frameworks have been focus on ad hoc networks. A general model was proposed in [2] to understand the cooperation among nodes in a mobile ad hoc network. The authors in [7] utilized the theory of imperfect private monitoring for the dynamic Bertrand oligopoly and derived conditions for collusive packet forwarding, truthful routing and packet acknowledgments in a multi-hop environment. In [12], the authors proposed a credit-based secure incentive mechanism in mobile ad hoc networks which is immune to various attacks and is of low communication overhead. Similarly, the work in [5] propose a secure and objective reputation-based incentive scheme where the reputation of a node is quantified by objective measures. An asymmetric cooperative caching mechanism was proposed in [13] which reduces not only the overhead of data copying but also the end-to-end delay. The authors in [14] proposed a cooperation-optimal protocol consisting of a routing protocol and a forwarding protocol and demonstrated that the protocols provide incentives for nodes to forward packets.

The emerging application of VoD services have also been implemented in ad hoc wireless networks. The authors in [9] designed a novel system that provides video-on-demand services to mobile ad hoc clients, which allows the clients to access video data anytime anywhere. Some recent works [3], [4], [6] have been focusing on various technical aspects of ad hoc wireless VoD systems. However, we have not found any incentive model for the specific application of VoD in ad hoc wireless environment. Our previous works [10], [11] focused on VoD incentives in a wired P2P environment, which differs from ad hoc wireless network although they share some common features. In this paper, we adopt similar ideas to design and analyze incentive mechanism in ad hoc wireless VoD systems.

VII. Conclusion

In this paper, we focus on the incentive mechanism design and game theoretic analysis in a wireless ad hoc environment. We design a reward based incentive scheme and analyze the interaction between the content provider and nodes using a Stackelberg game. We show the stability and efficiency of the Stackelberg equilibrium, and use repeated game models to analyze the threatening strategy and cheating prevention mechanisms. Our paper provides some important insights for incentive design principles in ad hoc wireless VoD systems.

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