Name:

Student ID:

Let MIN and MAX be the minimum and maximum of three independent samples of a Uniform(a, b) random variable. What is the largest p for which we can say that (MIN, MAX) is a p-confidence interval for the mean?

Solution: The mean falls outside the interval MIN, MAX if and only if all three samples X_1, X_2, X_3 are smaller than the mean μ, or all three samples are larger than the mean. By independence of the samples,

$$\mathrm{P}(\mu < MIN) = \mathrm{P}(X_1 < \mu, X_2 < \mu, X_3 < \mu) = \mathrm{P}(X_1 < \mu)\,\mathrm{P}(X_2 < \mu)\,\mathrm{P}(X_3 < \mu) = (1/2)^3 = 1/8,$$

and by symmetry $\mathrm{P}(\mu > MAX)$ is also $1/8$. As the two events are disjoint,

$$\mathrm{P}(\mu < MIN \text{ or } \mu > MAX) = \mathrm{P}(\mu < MIN) + \mathrm{P}(\mu > MAX) = 1/8 + 1/8 = 1/4$$

and so $\mathrm{P}(MIN \leq \mu \leq MAX) = 1 - \mathrm{P}(\mu < MIN \text{ or } \mu > MAX) = 3/4$.