Name:

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The *range* of a sample is the difference between the largest and smallest value observed. You observe three independent samples of data that is equally likely to take the values 1, 2, and 3. Calculate the PMF of the sample range.

Solution: The sample range R can take possible values 0, 1, and 2. The event $R = 0$ happens when all three samples X_1, X_2, X_3 are equal, so

$$\mathrm{P}(R = 0) = \mathrm{P}(X_3 = X_2 | X_2 = X_1)\,\mathrm{P}(X_2 = X_1) = \mathrm{P}(X_3 = X_2)\,\mathrm{P}(X_2 = X_1) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}.$$

The event $R = 2$ happens when both values 0 and 2 are represented in the sample. Let's call these events Z and T. By inclusion-exclusion

$$\mathrm{P}(R \neq 2) = \mathrm{P}(Z^c \cup T^c) = \mathrm{P}(Z^c) + \mathrm{P}(T^c) - \mathrm{P}(Z^c \cap T^c) = 2 \cdot \left(\frac{2}{3}\right)^3 - \left(\frac{1}{3}\right)^3 = \frac{5}{9}$$

so that $\mathrm{P}(R = 2) = 4/9$ and so $\mathrm{P}(R = 1) = 1 - \mathrm{P}(R = 0) - \mathrm{P}(R = 2) = 4/9$. The PMF is

r	0	1	2
$\mathrm{P}(R = r)$	1/9	4/9	4/9