There are three types of (unfair) three-sided dice, each with face values `1`, `2`, and `3`. A type-i die rolls face value i with probability $1/2$ and each of the other two face values with probability $1/4$. An unknown die is rolled twice and you observe the outcome `12`. Assuming an equally likely prior, what is the posterior type of the die that was rolled?

Solution: Let Θ be the prior type and X_1, X_2, X_3 be the rolls. By Bayes's rule,

$$f_{\Theta|X_1X_2}(\theta|12) \propto f_{X_1|\Theta}(1|\theta) \cdot f_{X_2|\Theta}(2|\theta) \cdot f_{\Theta}(\theta)$$

which evaluates to $(1/2)(1/4)(1/3)$ when $\theta = 1$, $(1/4)(1/2)(1/3)$ when $\theta = 2$, and $(1/4)(1/4)(1/3)$ when $\theta = 3$. In words, types 1 and 2 are equally likely and each is twice as likely as type 3. Therefore the posterior PMF must be

θ	1	2	3
$f_{\Theta\|X_1X_2}(\theta\|12)$	2/5	2/5	1/5.