Practice questions

1. The reported depth of the Mariana Trench is 11,000 meters deep. Your alternative hypothesis is that it is at least 12,000 meters deep.

   (a) You drop a sensor whose output is a Normal($\mu, 500$) random variable where μ is the true depth. It reports 11,800 meters. What is the p-value for your hypothesis?

   (b) How many (independent) sensors do you need to drop in order to test your hypothesis with a 10% false positive and a 10% false negative probability?

2. The PDF of a random variable is either $f_0(x) = 1/2$ (H_0) or $f_1(x) = 1/(\pi \cdot \sqrt{1-x^2})$ (H_1), where $-1 < x < 1$.

   (a) For a given threshold $t > 0$, describe the set of values x for which $f_1(x)/f_0(x) \geq t$.

   (b) Use part (a) and the Neyman-Pearson lemma to design a test (for a single sample) with false positive probability 1/4.

   (c) What is the false negative probability of your test?

3. A random variable X is normal with mean $\mu_0 = 15$ and a variance that is either $\sigma_0^2 = 10$ (H_0) or $\sigma_1^2 = 15$ (H_1). We want to test H_0 against H_1, using five sample values x_1, x_2, x_3, x_4, x_5 and a test of the form

$$T = \begin{cases} +, & \text{if } x_1 + x_2 + x_3 + x_4 + x_5 \geq \gamma \\ -, & \text{if not.} \end{cases}$$

   (a) Determine the value of γ so that the probability of a false negative is 0.05.

   (b) What is the corresponding false positive probability?

   [Adapted from textbook problem BT 9.3.20]

4. You suspect that when humans type long "random" strings (sequences of `0`s and `1`s) they tend to avoid long consecutive blocks with the same value. To test your hypothesis you design the following experiment: Ask each of 100 subjects to write a random 10-bit string. Then count the number X of answers in which all four middle bits are identical (`0000` or `1111`).

   (a) If the answers were truly random, what kind of random variable would X be?

   (b) State the null hypothesis (based on part (a)) and your alternative hypothesis.

   (c) Design a test for your hypothesis with a 10% false positive error.