## Practice questions

- 1. In Lecture 4 we showed that the maximum likelihood estimator for the mean  $\lambda$  of a Poisson $(\lambda)$  random variable given that N occurrences are observed within a time unit is N.
  - (a) Now subdivide the time unit into 10 equal intervals and suppose that  $N_i$  occurrences are observed in the *i*-th interval. The  $N_i$  are then independent samples of a Poisson $(\lambda/10)$  random variable. What is the maximum likelihood estimator for  $\lambda$ ?
  - (b) Is the maximum likelihood estimator in part (a) biased or not?
  - (c) (for ESTR) Can you come up with a sufficient statistic for n samples of a Poisson( $\lambda$ ) random variable?
- 2. You have a coin that is either always heads  $(\theta = 1)$  or fair  $(\theta = 0)$ .
  - (a) What is the maximum likelihood estimator for  $\theta$  from n independent coin flips?
  - (b) What is the unbiased estimator for  $\theta$  from one coin flip? (There is only one.)
  - (c) (**Optional**) Among all unbiased estimators for  $\theta$  from *n* coin flips, which one has the smallest variance?
- 3. Let N be a single sample of a Geometric( $\theta$ ) random variable.
  - (a) What is the maximum likelihood estimator for  $\theta$  from N? [Adapted from textbook problem BT9.1.2]
  - (b) Is the estimator from part (a) biased? (**Hint:**  $x + x^2/2 + x^3/3 + \cdots = \ln 1/(1-x)$  for -1 < x < 1.)
- 4. You are given three samples of a  $\operatorname{Zig}(\theta)$  random variable, which has PDF

$$f(x) = \begin{cases} 2(x - \theta), & \text{when } \theta \le x \le \theta + 1\\ 0, & \text{otherwise.} \end{cases}$$

- (a) What is the expected value  $\mu$  of a Zig( $\theta$ ) random variable?
- (b) Come up with an unbiased estimator for  $\theta$  that depends only on the sample mean  $\overline{X}$ .
- (c) Repeat part (b) for the sample maximum MAX.
- (d) What are the variances of your estimator from part (b) and part (c)? (**Hint:** Argue that the variance should not depend on  $\theta$  and assume  $\theta = 0$  in the calculation.)

## Additional ESTR 2020 questions

5. You want to estimate the mean  $\mu$  of a Normal( $\mu$ , 1) random variable from 100 samples, but 10% of these samples can be corrupted in an arbitrary way. In this setting the sample mean can be wildly inaccurate. Two alternatives are the sample median and the clipped sample mean (sample mean after removing some fraction of outliers). How would you evaluate the relative accuracy of these two estimators (without any assumptions about the locations or values of the corrupted samples)?