

### Practice questions

1. In an exam question half of the students scored 5 points, a quarter scored 3 points, and the rest scored no points. You are trying to figure out the average score by sampling three random students (with repetition) and asking for their score.
  - (a) What is the PMF of the average score of the three sampled students?
  - (b) What is the probability that the sample mean is equal to the actual mean?
  - (c) What is the probability that the sample mean is within one point of the actual mean?
2. Let  $X_1, X_2, X_3$  be independent samples of an  $\text{Indicator}(1/4)$  random variable. Calculate the PMF of the (a) sample mean (b) sample variance (c) sample standard deviation and (d) sample maximum.
3. A food processing company packages honey in glass jars. The volume of honey (in millilitres) in a random jar is a  $\text{Normal}(\mu, 10)$  random variable for some unknown  $\mu$ .
  - (a) What is the PDF of the sample mean volume of six random jars?
  - (b) What is the probability that the sample mean in part (a) is within 3 millilitres of the true mean  $\mu$ ?
4. Take  $n = 100$  samples of an  $\text{Indicator}(0.01)$  random variable. Let  $\bar{X}$  be the sample mean.
  - (a) What is the probability that the sample mean  $\bar{X}$  is within 0.005 of the true mean  $\mu$ ?
  - (b) The Central Limit Theorem says that the event  $\mu - \epsilon \leq \bar{X} \leq \mu + \epsilon$  should have similar probability to  $-t \leq N \leq t$  for large  $n$ , a  $\text{Normal}(0, 1)$  random variable  $N$ , and a suitable choice of  $t$ . What is the probability predicted for the event in part (a)?

### Additional ESTR 2020 questions

5. The *support size* of a discrete random variable is the number of distinct values that it may take. How many samples do you need to calculate the support size of an unknown random variable, say with 50% probability, if you know (a) it takes at most  $K$  distinct values (b) it takes no value with probability less than  $1/K$ ?