Practice questions

1. Alice is "fifty percent sure" that Bob's email address is `352642@acme.com`. She runs a test by sending sending 100 emails to random recipients of the form *abcdef*`@acme.com`. 95% of the emails bounce as undeliverable. Alice then sends an email intended for Bob to `352642@acme.com` and it does not bounce. What is the (posterior) probability that she correctly guessed his email address? *[Adapted from textbook problem 8.1.1]*

2. After much dating experience, Romeo concludes that girls show up Exponential(Θ) hours late to a date, where Θ is a Uniform$(0, 1)$ random variable that describes a girl's type.

   (a) Juliet is 10 minutes late on their first date. What is Romeo's posterior PDF for Θ?

   (b) Juliet is 30 minutes late on their second date. What is Romeo's posterior PDF now?

3. The number of mahjong games played in a given family on the Lunar New Year can be modeled as a Geometric(Θ) random variable, where the value of the parameter Θ varies from family to family. Assume a Uniform$(0, 1)$ prior on Θ.

   (a) Suppose Bob's family played x_1 games on the 2020 festival, x_2 games in 2019, up to x_t games t years ago. Show that the posterior is a Beta(α, β) random variable. What are α and β?

   (b) Bob observed that $x_1 = 22$ and $x_2 = 41$. What is the expected number of games that his family will play this year?

4. You have a coin of an unknown probability of heads P. Your prior is that P is a Uniform$(0, 1)$ random variable.

   (a) The coin is flipped 10 times and 9 of the 10 flips are heads. What is the posterior probability that $P > 80\%$?

   (b) (**Optional**) A second coin, whose prior is also uniform and independent of the first coin, is flipped 10 times and all flips are heads. What is the probability that the second coin is more biased towards heads than the first one?

Additional ESTR 2020 questions

5. Come up with a probabilistic model with one or two parameters for the sizes of the messages in an email inbox (without attachments). Then calculate the posterior for the parameter(s) of interest based on some data. You can use this dataset which lists the sizes of the last several hundred emails in my inbox to get you started.