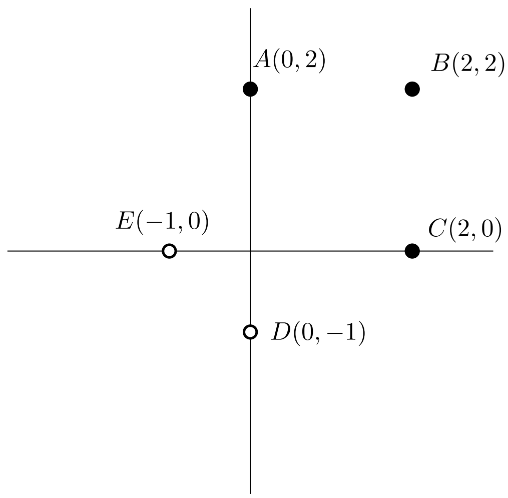
CMSC5724: Quiz 2

> Hand-write all your solutions on paper. Take a picture of the paper **together with** your CUHK student ID card, national ID card, or passport ID page. Upload the picture to Blackboard or email it to the instructor at *taoyf@cse.cuhk.edu.hk*. Your must do so before the deadline to be announced at the beginning of the quiz.

Problem 1 (50%). The following figure shows a set of 5 points. Use the Perceptron algorithm to find a line that (i) crosses the origin and (ii) separates the black points from the white ones. Recall that Perceptron starts with a vector $\boldsymbol{w} = \mathbf{0}$ and iteratively adjusts it using a violation point. You need to show the value of $\boldsymbol{w}$ after every adjustment.



Answer: Without loss of generality, assume that the black points have label 1 while the white ones have label -1. At the beginning, $\boldsymbol{w} = (0, 0)$. We use $\boldsymbol{A}$ to denote the vector form of A. Define $\boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ and $\boldsymbol{E}$ similarly.

Iteration 1. Since $\boldsymbol{A}$ does not satisfy $\boldsymbol{w} \cdot \boldsymbol{A} > 0$, update $\boldsymbol{w}$ to $\boldsymbol{w} + \boldsymbol{A} = (0, 0) + (0, 2) = (0, 2)$.

Iteration 2. Since $\boldsymbol{C}$ does not satisfy $\boldsymbol{w} \cdot \boldsymbol{A} > 0$, update $\boldsymbol{w}$ to $\boldsymbol{w} + \boldsymbol{C} = (0, 2) + (2, 0) = (2, 2)$.

Iteration 3. No more violation points. So we have found a separation line $2x + 2y = 0$.

Problem 2 (50%). Suppose that $\mathcal{H}$ is a set of classifiers, each mapping a point in $\mathbb{R}^d$ to $\{-1, 1\}$ for some integer d (which will not matter). Consider any subset $\mathcal{H}' \subseteq \mathcal{H}$. Prove: for any set P of points in $\mathbb{R}^d$, VC-dim$(P, \mathcal{H}') \leq$ VC-dim$(P, \mathcal{H})$.

Answer: Suppose that there exists a set P satisfying VC-dim$(P, \mathcal{H}') >$ VC-dim$(P, \mathcal{H})$. Then, there must be a subset S of P such that $|S| =$ VC-dim$(P, \mathcal{H}')$ and S is shattered by $\mathcal{H}'$. According to the fact $\mathcal{H}' \subseteq \mathcal{H}$, we know S is also shattered by $\mathcal{H}$. However, this implies that VC-dim$(P, \mathcal{H}') \leq$ VC-dim$(P, \mathcal{H})$, thus causing a contradiction.