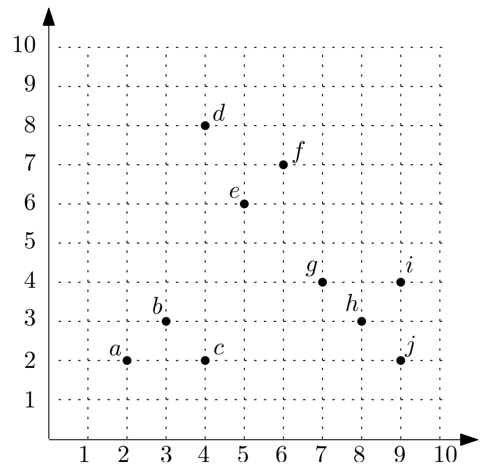
# CMSC5724: Exercise List 8

**Problem 1.** Consider the execution of the $k$-center algorithm we discussed in the class on the following set $P$ of points:




Suppose that $k = 3$ (i.e., we want to find 3 centers), and that the first center has been (randomly) decided to be $f$. Show what are the second and third centers found by the algorithm. The distance metric is Euclidean distance.

**Problem 2.** Let $P$ be the set of points in Problem 1. What is the geometric center of the set $\{c, e, g\}$?

**Problem 3.** Let $P$ be the set of points in Problem 1. Apply the $k$-means algorithm on $P$ with $k = 3$ under Euclidean distance. Assume that the algorithm selects a set $S = \{c, g, h\}$ as the initial centroids. Recall that (i) the algorithm updates $S$ iteratively, and (ii) the cost of $S$ is defined to be $\phi(S) = \sum_{p \in P}(d_S(p))^2$ where $d_S(p) = \min_{q \in S} dist(p, q)$.

- Given the content of $S$ after each iteration until the algorithm terminates.
- Show the value of $\phi(S)$ after every iteration.

**Problem 4.** The goal of this problem is for you to understand why it suffices to consider a finite number of possible solutions to the $k$-means problem (recall that this was needed to argue that the algorithm terminates).

Consider the $k$-means problem defined in the lecture notes with $k = 2$. Suppose that we have a set $P$ of $n$ points in $\mathbb{R}^2$ (for simplicity, we assume that the dimensionality is 2). The goal is to find centroid points $c_1, c_2$ in $\mathbb{R}^2$ to minimize $\sum_{p \in P}(d(p))^2$, where $d(p) = \min_{i=1}^{2} dist(p, c_i)$, with $dist$ representing Euclidean distance. Design an algorithm to solve this problem in $O(2^n \cdot n)$ time.