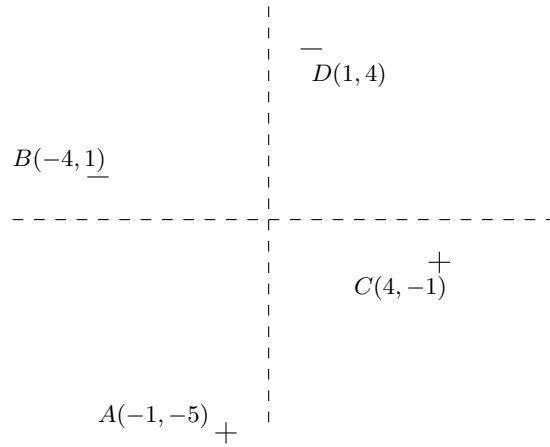


## CMSC5724: Exercise List 5

Answer all the problems below based on the following set  $P$  of points  $A, B, C$  and  $D$ :



where “+” represents label 1 and “-” represents label  $-1$ .

**Problem 1.** What is the margin of the separation line  $\ell : -x - 5y = 0$ ?

**Problem 2.** Run Margin Perceptron on  $P$  with  $\gamma_{guess} = 0.1$ , and give the equation of the line that is maintained by the algorithm at the end of each iteration.

**Problem 3.** Same as the previous problem but with  $\gamma_{guess} = 4/\sqrt{26}$ .

**Problem 4.** Give an instance of quadratic programming to find an origin-passing separation plane with the maximum margin.

**Problem 5.** Consider the following instance of quadratic programming in  $\mathbb{R}^d$ :

$$\begin{aligned} & \text{minimize } |\mathbf{w}| \text{ subject to} \\ & \mathbf{w} \cdot \mathbf{p}_i \geq 1 \text{ for each } i \in [1, n] \end{aligned}$$

where  $\mathbf{p}_1, \dots, \mathbf{p}_n$  are  $n$  given points in  $\mathbb{R}^d$ . Prove: if an optimal  $\mathbf{w}$  exists, there must exist at least one  $i \in [1, n]$  such that  $\mathbf{w} \cdot \mathbf{p}_i = 1$ .

**Problem 6.** Let  $\gamma_{opt}$  be the maximum margin of an origin-passing separation plane on a set  $P$  of points. Denote by  $R$  the largest distance from a point in  $P$  to the origin.

Suppose that, given a value  $\gamma$ , margin Perceptron ensures the following:

- if it terminates, it definitely returns a separation plane with margin at least  $\alpha \cdot \gamma$ , where  $\alpha$  is an arbitrary constant less than 1;
- if  $\gamma \leq \gamma_{opt}$ , it definitely terminates after at most  $c \cdot R^2/\gamma^2$  corrections, for some constant (which depends on  $\alpha$ ).

Design an algorithm to find a separation plane with margin at least  $\alpha \cdot \beta \cdot \gamma_{opt}$  after  $O(R^2/\gamma_{opt}^2)$  corrections in total, where  $\beta$  can be any constant less than 1.