CSCI2100: Special Exercise Set 4

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Problem 1. Recall that, in merge sort, the merging step combines two sorted lists A_1, A_2 into one sorted list A. Suppose that A_1 and A_2 are $(1, 8, 17, 23, 35, 83)$ and $(3, 5, 15, 28, 56)$, respectively. Describe the content of A right before the moment when 23 enters A.

Problem 2. For the k-selection problem, suppose that the input is an array of 12 elements: (58, 23, 98, 83, 32, 24, 18, 45, 85, 91, 2, 34). Recall that our randomized algorithm first selects a number v, and then recursively solves the problem on a smaller array. Suppose that $v = 34$ and $k = 10$. What is the smaller array that we will recurse into?

Problem 3 (Textbook Exercise 9.3-5). The *median* of a set S of n elements is the $\lfloor n/2 \rfloor$ smallest element in S. Suppose that you are given a deterministic algorithm for finding the median of S (stored in an array) in $O(n)$ worst-case time. Using this algorithm as a black box, design another deterministic algorithm for solving the k-selection problem (for any $k \in [1, n]$) in $O(n)$ worst-case time.

Problem 4. Let S be a set of n integers, and k_1, k_2 arbitrary integers satisfying $1 \leq k_1 \leq k_2 \leq n$. Suppose that S is given in an array. Give an $O(n)$ expected time algorithm to report *all* the integers whose ranks in S are in the range $[k_1, k_2]$. Recall that the rank of an integer v in S equals the number of integers in S that are at most v.

Problem 5. Let S be a set of n integers given in an array. Describe a deterministic algorithm to find the 100 largest integers in S in $O(n)$ worst-case time.