Exercises: Matrix Basic Operations and Gauss Elimination

Problem 1. Let $\boldsymbol{A}$ be a square $n \times n$ matrix, and $\boldsymbol{I}$ an identity $n \times n$ matrix. Prove $\boldsymbol{AI} = \boldsymbol{A}$, and $\boldsymbol{IA} = \boldsymbol{A}$.

Problem 2. Calculate $\boldsymbol{AB}$, $\boldsymbol{BA}$, and $\boldsymbol{A}^T\boldsymbol{B}^T$, where

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ -1 & -2 & 1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Problem 3. $\boldsymbol{A}, \boldsymbol{B}$, and $\boldsymbol{C}$ are $m \times n$, $n \times p$, and $p \times q$ matrices. Prove: $(\boldsymbol{ABC})^T = \boldsymbol{C}^T\boldsymbol{B}^T\boldsymbol{A}^T$.

Problem 4. What is $\boldsymbol{A}^T$ if $\boldsymbol{A}$ is (i) symmetric, and (ii) anti-symmetric?

Problem 5. $\boldsymbol{A}$ and $\boldsymbol{B}$ are both $n \times n$ symmetric matrices. Prove: AB is symmetric if and only if $AB = BA$.

Problem 6. Consider the following recurrence for $i \geq 1$:

$$\boldsymbol{x_{i+1}} = \boldsymbol{Ax_i}$$

where $\boldsymbol{A}$ is an 3×3 matrix, and $\boldsymbol{x_i}$ and $\boldsymbol{x_{i+1}}$ are 3×1 matrices. Knowing:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \text{ and } \boldsymbol{x_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

what is the value of $\boldsymbol{x_3}$?

Problem 7. Convert the following matrix into row echelon form with elementary row operations:

$$\begin{bmatrix} 0 & 3 & 1 & 1 \\ 0 & 0 & 5 & 5 \\ 1 & -1 & 3 & 3 \\ 3 & 3 & -7 & -7 \end{bmatrix}$$

Problem 8. Solve the following linear system with Gauss Elimination

$$\begin{aligned} 4y + 3z &= 8 \\ 2x - z &= -2 \\ x + 2z &= 5. \end{aligned}$$

Problem 9. Decide if the following linear system is consistent.

$$\begin{aligned} 4y + 3z &= 8 \\ 2x - z &= -2 \\ x + 2y + z &= 3. \end{aligned}$$

If it is, give all the solutions to the system.