# CSCI3160: Special Exercise Set 8

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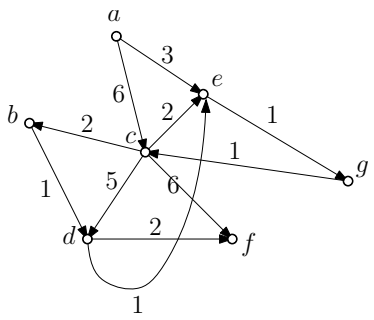
**Problem 1.** Consider the weighted directed graph below.




Suppose that we run Dijkstra's algorithm starting from vertex $a$. Recall that the algorithm relaxes the outgoing edges of every other vertex in turn. Give the order of vertices by which the algorithm relaxes their edges.

**Problem 2.** Consider a weighted directed graph $G = (V, E)$. Let $s$ and $v$ be two distinct vertices in $G$. Denote by $IN(v)$ the set of in-neighbors of $v$ (i.e., recall that a vertex $u$ is an *in-neighbor* of $v$ if $(u, v)$ is an edge in $E$). Prove:

$$spdist(s, v) \leq \min_{u \in IN(v)} \{spdist(s, u) + w(u, v)\}.$$

where $spdist(x, y)$ is the shortest path distance from vertex $x$ to vertex $y$, and $w(u, v)$ is the weight of the edge $(u, v)$.

**Problem 3.** In the context of Problem 2, prove:

$$spdist(s, v) \geq \min_{u \in IN(v)} \{spdist(s, u) + w(u, v)\}.$$

**Problem 4.** Let $G = (V, E)$ be a weighted directed graph. Give an algorithm to compute the shortest path distances between all pairs of vertices. Your algorithm should finish in $O(|V|(|V| + |E|) \log |V|)$ time.

**Problem 5.** Adapt Dijkstra's algorithm to solve the SSSP problem on a weighted undirected graph.