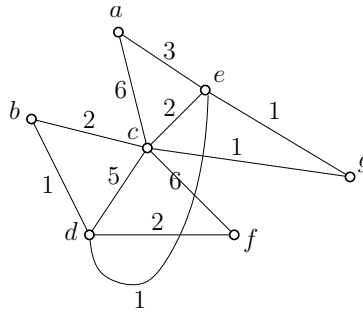# CSCI3160: Special Exercise Set 4

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Problem 1. Let T be a tree. Prove: for any two distinct nodes u, v in the tree, there exists one and exactly one simple path from u to v (a simple path is a path where no vertex appears twice).

Problem 2. Consider the weighted undirected graph below.

Suppose that we run Prim's algorithm to find a minimum spanning tree (MST) of this graph. Explain in what order does the algorithm insert edges into the tree.

Problem 3. Consider again the execution of Prim's algorithm in Problem 2. Indicate how the cross edges (i.e., "extension edges" as defined in the tutorial) change as Prim's algorithm runs. That is, point out the cross edges after the first edge is picked, the cross edges after the second edge is picked, and so on.

Problem 4 (The Cut-Property) Let $G = (V, E)$ be an undirected graph where each edge in E is associated with a positive integer weight. We assume that all the edges in E have *distinct* weights. Consider any non-empty $S \subset V$, namely, S is a non-empty subset of V but does not contain at least one element in V. An edge $\{u, v\}$ in E is a *cross edge* if $u \in S$ but $v \notin S$. Prove: the cross edge with the minimum weight must belong to the MST of G.