

## CSCI3610: Special Exercise Set 1

**Problem 1.** Explain how to implement the operation  $x \bmod y$  in  $O(1)$  time where  $x$  and  $y$  are positive integers.

**Problem 2.** For the  $k$ -selection problem, suppose that the input is an array of 12 elements: (58, 23, 98, 83, 32, 24, 18, 45, 85, 91, 2, 34). Recall that our randomized algorithm first selects a number  $v$ , and then recursively solves the problem on a smaller array. Suppose that  $v = 34$  and  $k = 10$ . What is the smaller array that we will recurse into?

**Problem 3 (Textbook Exercise 9.3-5).** The *median* of a set  $S$  of  $n$  elements is the  $\lfloor n/2 \rfloor$  smallest element in  $S$ . Suppose that you are given a deterministic algorithm for finding the median of  $S$  (stored in an array) in  $O(n)$  worst-case time. Using this algorithm as a black box, design another deterministic algorithm for solving the  $k$ -selection problem (for any  $k \in [1, n]$ ) in  $O(n)$  worst-case time.

**Problem 4.** Let  $S$  be a set of  $n$  integers, and  $k_1, k_2$  arbitrary integers satisfying  $1 \leq k_1 \leq k_2 \leq n$ . Suppose that  $S$  is given in an array. Give an  $O(n)$  expected time algorithm to report *all* the integers whose ranks in  $S$  are in the range  $[k_1, k_2]$ . Recall that the rank of an integer  $v$  in  $S$  equals the number of integers in  $S$  that are at most  $v$ .

**Problem 5\* (Textbook Exercise 9-2).** We are given an array that stores a set  $S$  of  $n$  distinct integers. Set  $W = \sum_{e \in S} e$ . Describe an algorithm to find the element  $e^* \in S$  that makes both of the following hold:

- $\sum_{e < e^*} e < W/2$
- $\sum_{e > e^*} e \leq W/2$ .

Your algorithm should finish in  $O(n)$  time ( $O(n)$  expected time is acceptable).

(Hint: First convince yourself that such  $e^*$  is unique, and then adapt the  $k$ -selection algorithm).