

## CSCI3160: Regular Exercise Set 4

Prepared by Yufei Tao

**Problem 1.** Recall that a *tree* is a connected graph without cycles. Prove:

- Every tree has at least a leaf node, i.e., a node with degree 1 (i.e., a node incident to only one edge).
- Every tree with  $n$  nodes has precisely  $n - 1$  edges.

**Problem 2.** Let  $S$  be a set of integer pairs of the form  $(id, v)$ . We will refer to the first field as the *id* of the pair, and the second as the *key* of the pair. Design a data structure that supports the following operations:

- Insert: add a new pair  $(id, v)$  to  $S$  (you can assume that  $S$  does not already have a pair with the same id).
- Delete: given an integer  $t$ , delete the pair  $(id, v)$  from  $S$  where  $t = id$ , if such a pair exists.
- DeleteMin: remove from  $S$  the pair with the smallest key, and return it. .

Your structure must consume  $O(n)$  space, and support all operations in  $O(\log n)$  time where  $n = |S|$ .

**Problem 3.** Prove: in a weighted undirected graph  $G = (V, E)$  where all the edges have distinct weights, the minimum spanning tree (MST) is unique.

**Problem 4.** Describe how to implement the Prim's algorithm on a graph  $G = (V, E)$  in  $O((|V| + |E|) \cdot \log |V|)$  time.