CSCI3160: Regular Exercise Set 2

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Problem 1 (Faster Algorithm for Finding the Number of Crossing Inversions). Let S_1 and S_2 be two disjoint sets of n integers. Assume that S_1 is stored in an array A_1, and S_2 in an array A_2. Both A_1 and A_2 are sorted in ascending order. Design an algorithm to find the number of such pairs (a, b) satisfying all of the following conditions: (i) $a \in S_1$, (ii) $b \in S_2$, and (iii) $a > b$. Your algorithm must finish in $O(n)$ time (we gave an $O(n \log n)$-time algorithm in the class).

Problem 2. Give an $O(n \log n)$-time algorithm to solve the dominance counting problem discussed in the class. (Hint: Require the $n/2$ points on each of side of the split line to be sorted after recursion.)

Problem 3 (Section 4.1 of the Textbook). Let A be an array of n integers (A is not necessarily sorted). Each integer in A may be positive or negative. Given i, j satisfying $1 \leq i \leq j \leq n$, define *sub-array* $A[i : j]$ as the sequence $(A[i], A[i + 1], ..., A[j])$, and the *weight* of $A[i : j]$ as $A[i] + A[i + 1] + ... + A[j]$. For example, consider $A = (13, -3, -25, 20, -3, -16, -23, 18)$; $A[1 : 4]$ has weight 5, while $A[2 : 4]$ has weight -8.

1. Give an algorithm to find a sub-array of with the largest weight, among all sub-arrays $A[i : j]$ with $j = n$. Your algorithm must finish in $O(n)$ time.
2. Give an algorithm to find a sub-array with the largest weight in $O(n \log n)$ time (among *all* the possible sub-arrays).

Problem 4. In the class, we explained how to multiply two $n \times n$ matrices in $O(n^{2.81})$ time when n is a power of 2. Explain how to ensure the running time for any value of n.