香港中文大學 The Chinese University of Hong Kong
教育學院 Faculty of Education
香港教育研究所 Hong Kong Institute of Educational Research
學生能力國際評估計劃－香港中心 Hong Kong Centre for International Student Assessment

PISA 2009 學校講座 School Seminar

Hong Kong Students' Performance in Mathematical Literacy

黃家樂
WONG Ka Lok

14 December 2010

Mathematical Literacy in PISA

Definition and its distinctive features

"an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (OECD, 2009, p.84)

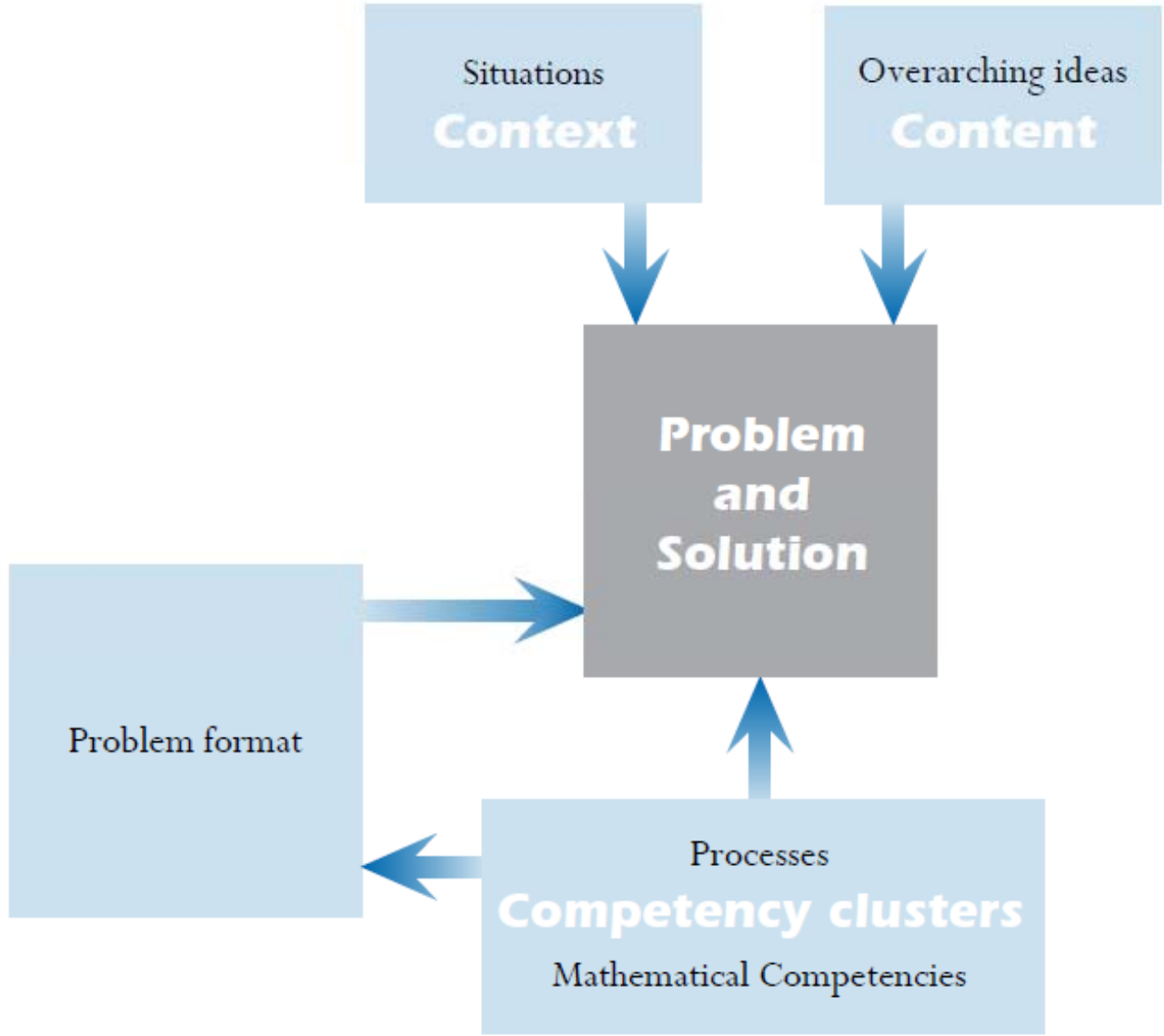
not limited to knowledge of mathematical terminologies, facts as well as skills in carrying out mathematical operations and standard procedures

more concerned with "the ability of students to analyse, reason and communicate ideas effectively as they pose, formulate, solve and interpret mathematical problems in a variety of situations" (OECD, 2009, p.84)

Mathematical Literacy in PISA

Mathematical literacy is related to wider, functional use of mathematics. ***Engagement with mathematics*** includes the ability to recognise and formulate mathematical problems in various situations.	
Knowledge Domain (Content)	Clusters of relevant mathematical areas and concepts: • ***Quantity*** • ***Space and shape*** • ***Change and relationships*** • ***Uncertainty***
Competencies involved (Processes)	Competency clusters define skills needed for using mathematics: • ***Reproduction*** **(simple mathematical operations)** • ***Connections*** **(bringing together ideas to solve problems)** • ***Reflection*** **(wider mathematical thinking)**
Context and situation	Various areas of application of mathematics, focusing on uses in different settings: • ***Educational*** • ***Intra-Mathematical*** • ***Occupational*** • ***Personal*** • ***Public*** • ***Scientific***

Mathematical Literacy in PISA



Components of the mathematics domain, taken from OECD (2009, p.90)

HK Students' Performance in Science, Mathematics and Reading from PISA2000+, 2003, 2006 to 2009

	Science		Mathematics		Reading	
Year	Mean	S.E.	Mean	S.E.	Mean	S.E.
2000+	(541)	3.0	(560)	3.3	525	2.9
2003	(539)	4.3	550	4.5	510	3.7
2006	542	2.5	547	2.7	536	2.4
2009	549	2.8	555	2.7	533	2.1

Performance in Mathematical Literacy
of Participating Countries/Regions in PISA 2009

Country/Region	Mean	S.E.	Significance
Shanghai-China	600	(2.8)	▲
Singapore	562	(1.4)	▲
Hong Kong-China	555	(2.7)	--
Korea	546	(3.9)	O
Chinese Taipei	543	(3.4)	▼
Finland	541	(2.2)	▼
Liechtenstein	536	(4.1)	▼
Switzerland	534	(3.3)	▼
Japan	529	(3.4)	▼
Canada	527	(1.6)	▼
Netherlands	526	(4.7)	▼
Macao-China	525	(0.9)	▼
…	…	…	
OECD Average	496	(0.5)	▼

Remarks

▲ denotes score t hat is significantly higher than that of Hong Kong

O denotes score t hat is not significantly different from that of Hong Kong

▼ denotes score t hat is significantly lower than that of Hong Kong

Performance in Mathematical Literacy
of Participating Countries/Regions in PISA 2009

Country/Region	Mean	S.E.	Significance
OECD Average	496	(0.5)	▼
...	...	...	
Argentina	388	(4.1)	▼
Jordan	387	(3.7)	▼
Brazil	386	(2.4)	▼
Colombia	381	(3.2)	▼
Albania	377	(4.0)	▼
Tunisia	371	(3.0)	▼
Indonesia	371	(3.7)	▼
Qatar	368	(0.7)	▼
Peru	365	(4.0)	▼
Panama	360	(5.2)	▼
Kyrgyzstan	331	(2.9)	▼

Remarks

▲ denotes score t hat is significantly higher than that of Hong Kong

O denotes score t hat is not significantly different from that of Hong Kong

▼ denotes score t hat is significantly lower than that of Hong Kong

Mathematical Proficiency Levels

Score Range of the Mathematical Proficiency Levels	
Proficiency Levels	**Lower Score Limit**
6	**669.3**
5	**607.0**
4	**544.7**
3	**482.4**
2	**420.1**
1	**357.8**
Below 1	**Below 357.8**

Proficiency Levels 1 – 6

- General ability of an individual in mathematics and related areas, and thus his/her ***prospects and capacity to participate fully in the society***

- Also implications for the role that the country will play in the advancing technological world, i.e. ***the country's competitiveness***

Figure 6.16

Summary descriptions of the six proficiency levels in mathematics

Level	Lower score limit	What students can typically do
6	669.3	At Level 6 students can conceptualise, generalise, and utilise information based on their investigations and modelling of complex problem situations. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situations.
5	607.0	At Level 5 students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.
4	544.7	At Level 4 students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic ones, linking them directly to aspects of real-world situations. Students at this level can utilise well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions.
3	482.4	At Level 3 students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications reporting their interpretations, results and reasoning.
2	420.1	At Level 2 students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.
1	357.8	At Level 1 students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

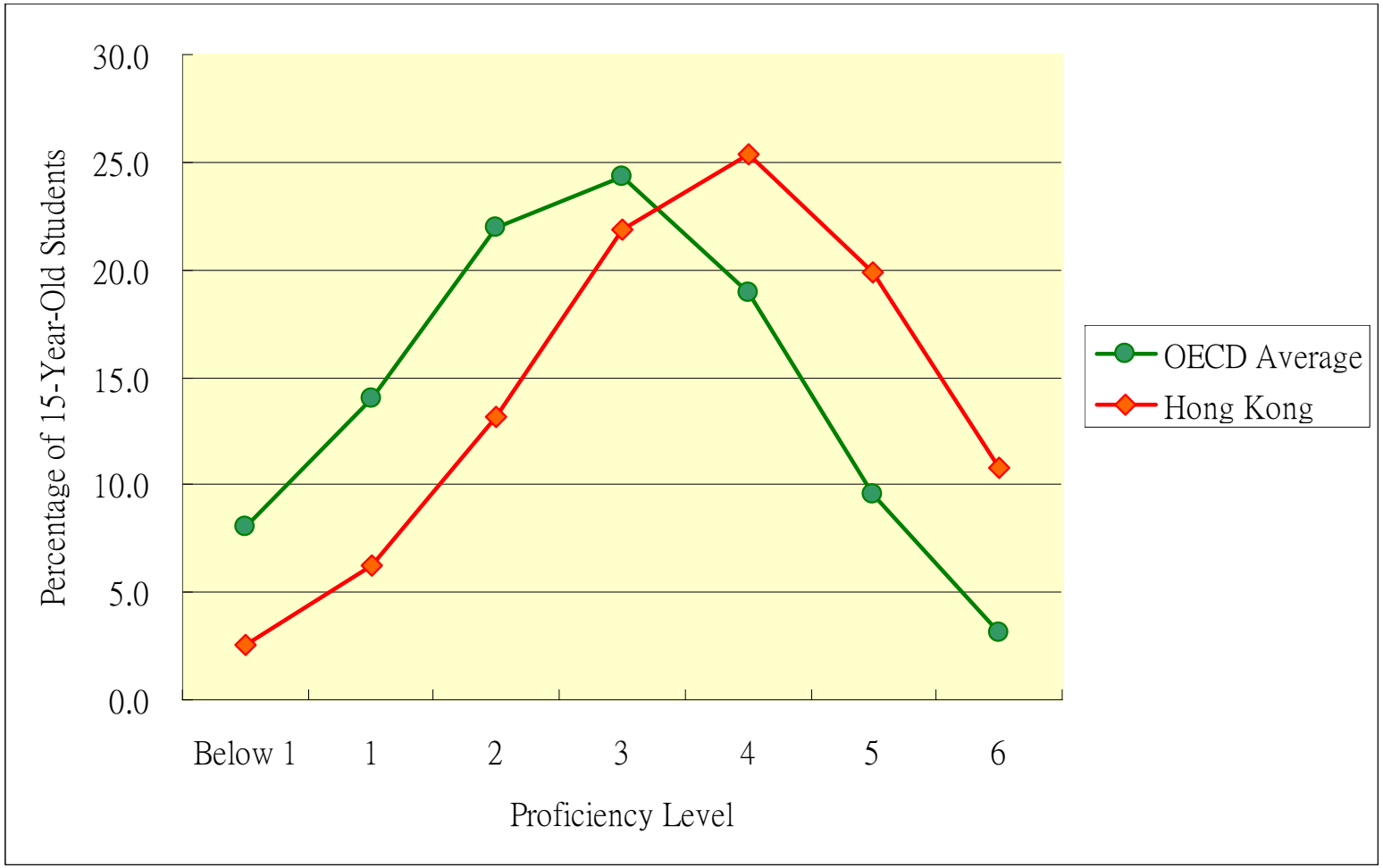
Details can be found in OECD (2007) *PISA 2006: Science Competencies for Tomorrow's World, Volume 1* (p.312), available at http://www.pisa.oecd.org/.

Percentage of Students
at each Level of Proficiency
on the scale of mathematical literacy
Hong Kong vs OECD Average

	Hong Kong	OECD Average	Difference (HK – OECD)
Level 6	10.8	3.1	**+7.7 *****
Level 5	19.9	9.6	**+10.3 *****
Level 4	25.4	18.9	**+6.5 *****
Level 3	21.9	24.3	–2.4 **
Level 2	13.2	22.0	–8.8 ***
Level 1	6.2	14.0	–7.8 ***
Below Level 1	2.6	8.0	–5.4 ***

*** / ** Difference is significant at the 0.001 / 0.01 level.

Percentage of Students
at each Level of Proficiency
on the scale of mathematical literacy in PISA 2009
Hong Kong vs OECD Average



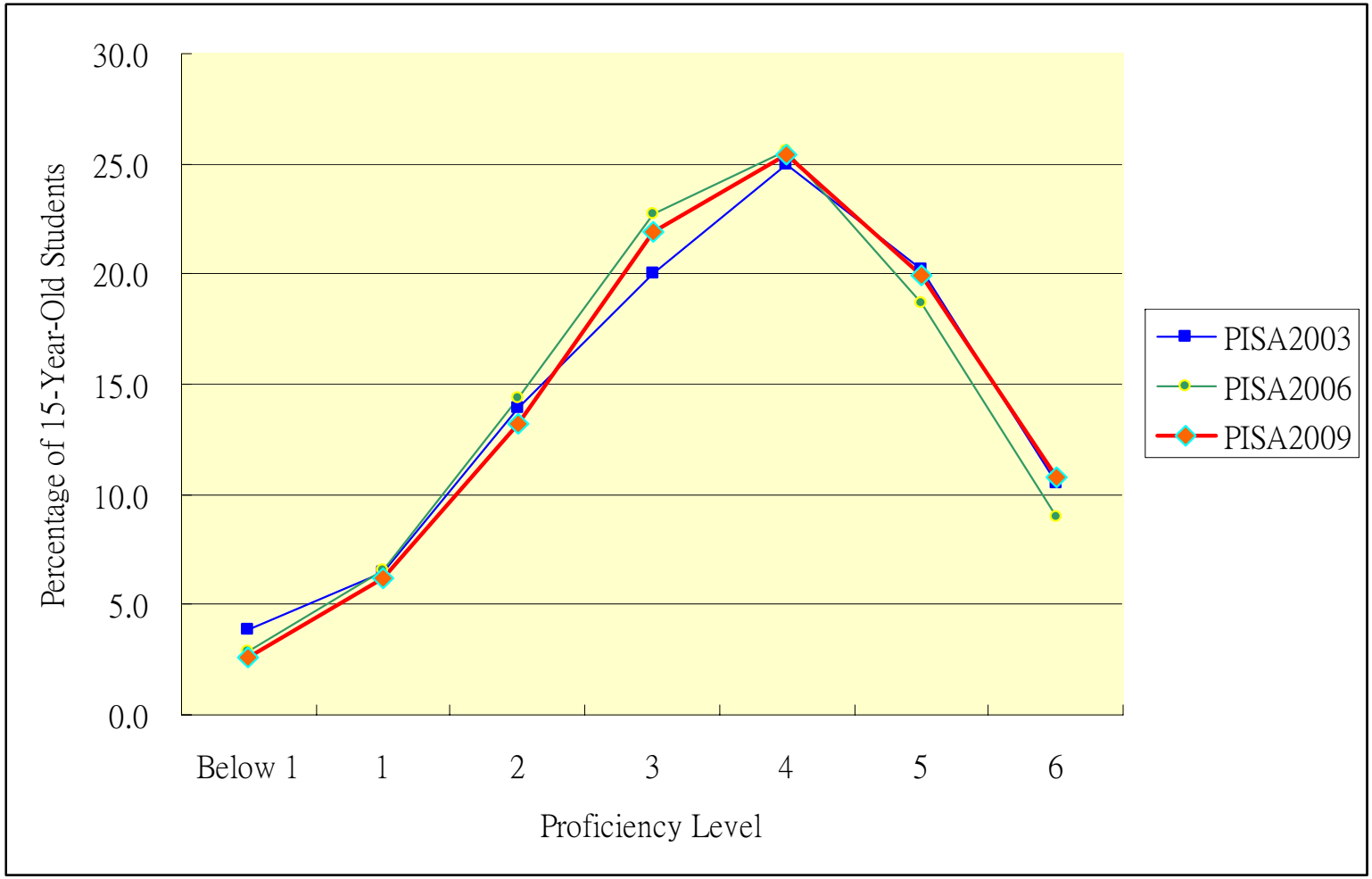
Percentage of Hong Kong Students
at each Level of Proficiency
on the scale of mathematical literacy

	PISA 2003	PISA 2006		PISA 2009	
Level 6	10.5	9.0	(–1.5)	**10.8**	(+1.8)
Level 5	20.2	18.7	(–1.4)	**19.9**	(+1.2)
Level 4	25.0	25.6	(+0.6)	**25.4**	(–0.2)
Level 3	20.0	22.7	(+2.8)	**21.9**	(–0.8)
Level 2	13.9	14.4	(+0.5)	**13.2**	(–1.2)
Level 1	6.5	6.6	(+0.1)	**6.2**	(–0.4)
Below Level 1	3.9	2.9	(–1.0)	**2.6**	(–0.4)

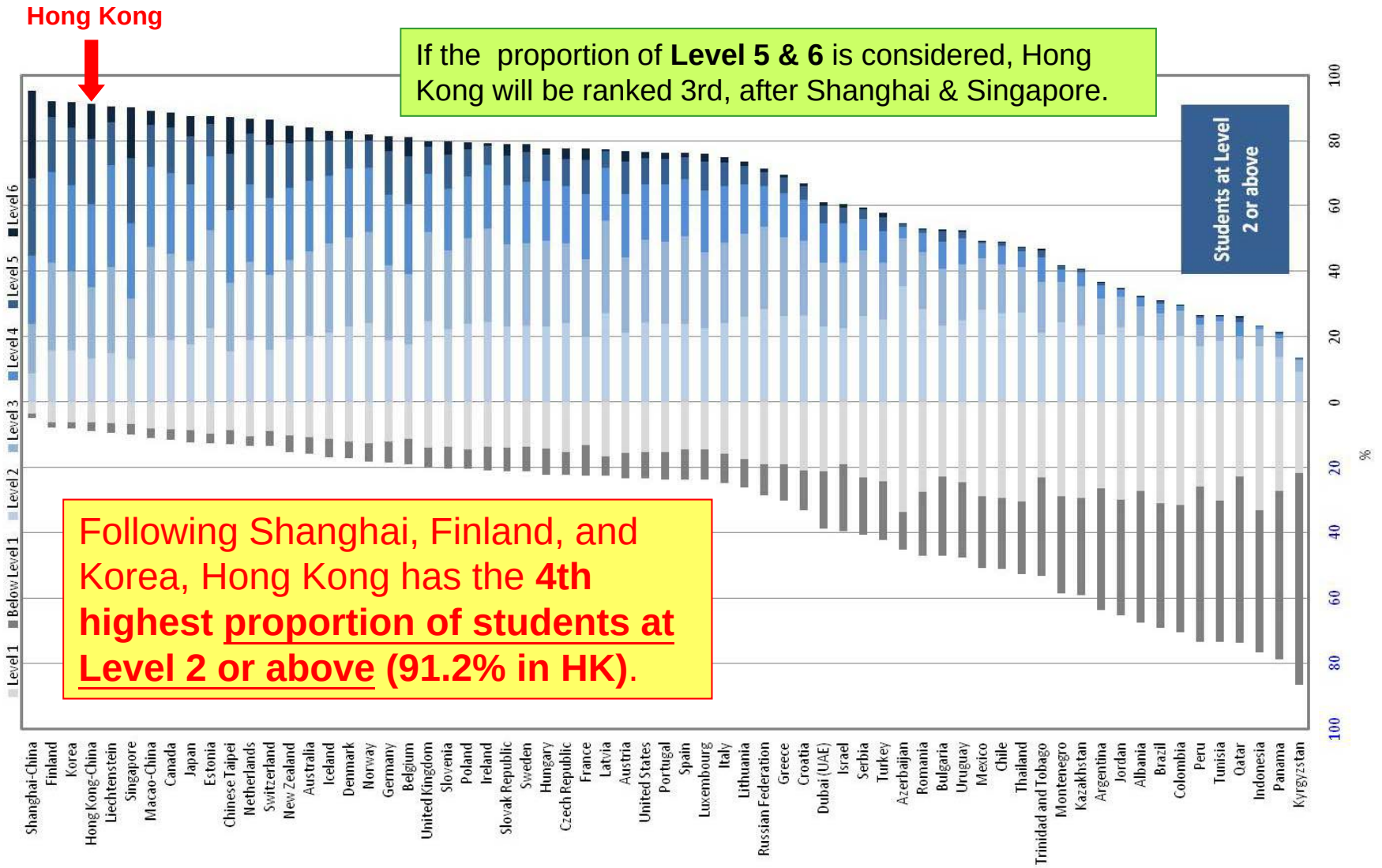
Numbers in brackets are DIFFERENCES (expressed by percentage points) from the corresponding percentages in the *previous* PISA cycle.

The differences at all Levels of Proficiency between two successive years are **statistically insignificant**.

Percentage of Students
at each Level of Proficiency
on the scale of mathematical literacy in HKPISA
from 2003 to 2006, and to 2009



Percentage of students
at each LEVEL OF PROFICIENCY
on the scale of mathematical literacy in PISA 2009



Percentage of Students at Proficiency Level 5 or Above
Countries/Regions with a Total of More Than 20%

Country/Region	Percentage at Level 5 (606.99 – 669.30)	Percentage at Level 6 (above 669.30)	Total Percentage at Level 5 or Above
Shanghai-China	23.8%	26.6%	50.4%
Singapore	20.0%	15.6%	35.6%
Hong Kong	19.9%	10.8%	30.7%
Chinese-Taipei	17.2%	11.3%	28.5%
Korea	17.7%	7.8%	25.5%
Switzerland	16.3%	7.8%	24.1%
Finland	16.7%	4.9%	21.6%
Japan	14.7%	6.2%	20.9%
Belgium	14.6%	5.8%	20.3%
OECD countries	*9.6%*	*3.1%*	*12.7%*

Percentage of Correct Answers (1)

Hong Kong and the OECD Average

Distribution of Items …	Number of items	Percent Correct: Hong Kong	Percent Correct: OECD Average
by Mathematical Strand (content)			
Algebra	1	**23**	7
Discrete Mathematics	2	**62**	42
Functions	2	**53**	44
Geometry	8	**53**	40
Number	11	**67**	56
Probability	2	**68**	60
Statistics	9	**58**	46
by "overarching ideas"			
Change and relationships	9	**56**	44
Quantity	11	**65**	53
Space and Shape	8	**53**	40
Uncertainty	7	**61**	49

Percentage of Correct Answers (2)

Hong Kong and the OECD Average

Distribution of Items …	Number of items	Percent Correct: Hong Kong	Percent Correct: OECD Average
by Competency Class (process)			
Reproduction	9	**74**	65
Connection	18	**57**	43
Reflection	8	**48**	35
by Situation (context)			
Educational	4	**59**	53
Intra-Mathematical	1	**18**	11
Occupational	1	**39**	28
Personal	4	**77**	74
Public	13	**63**	45
Scientific	12	**54**	43

Percentage of Correct Answers (3)

Hong Kong and the OECD Average

Distribution of Items …	Number of items	Percent Correct	
		Hong Kong	OECD Average
by Item Format			
Multiple-Choice	9	**74**	62
Complex Multiple-Choice	7	**59**	48
Closed-Constructed Response	3	**59**	51
Open-Constructed Response	8	**46**	30
Short Response	8	**56**	44

On ***every*** **dimension/category described by the PISA assessment framework**, the **percentage of correct answers** of Hong Kong 15-year-old students is **HIGHER** than that of the OECD Average.

Comparison of Percentile Scores between Hong Kong and OECD Average in Mathematical Literacy

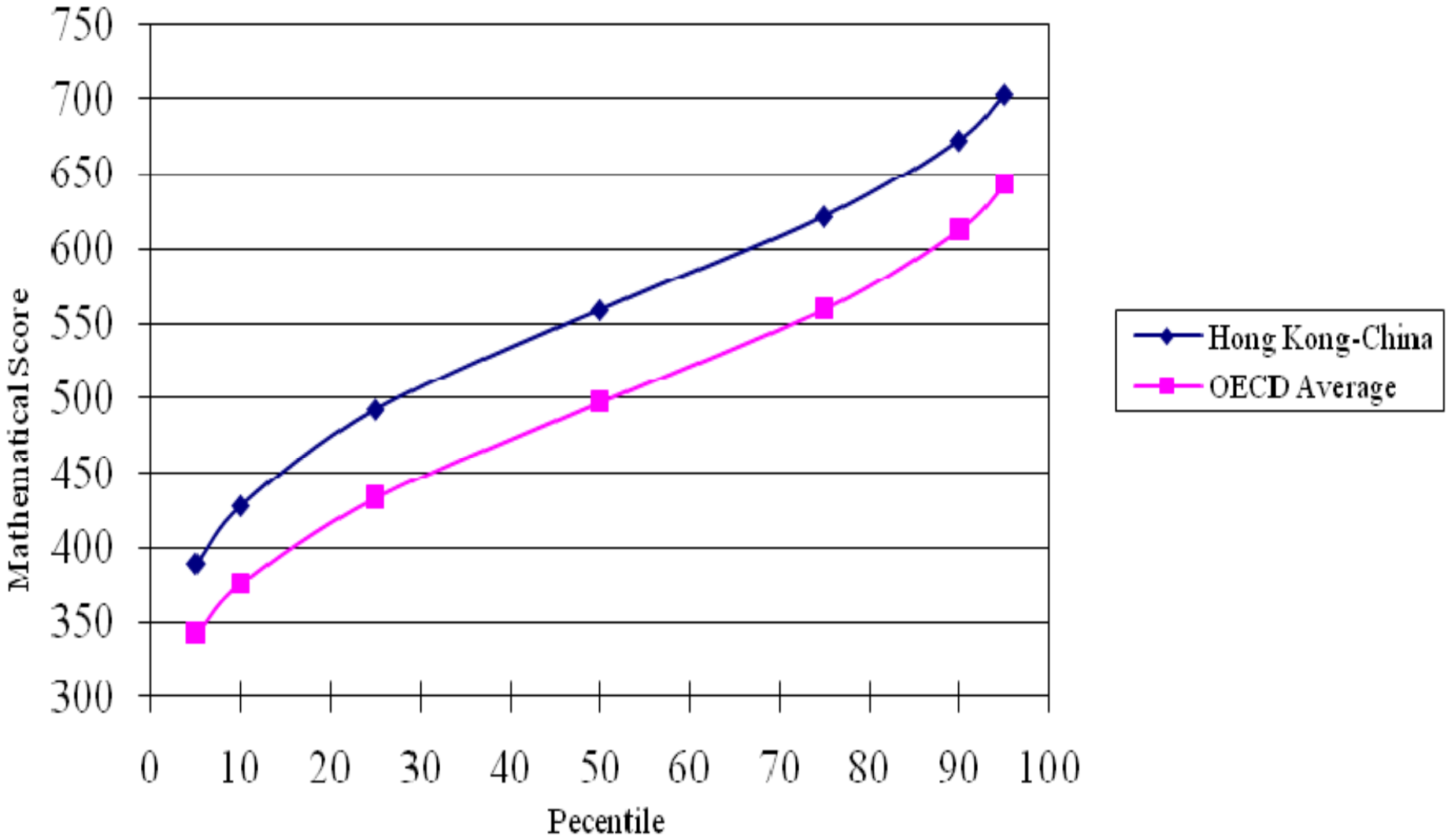
at Different Percentiles

Percentile	Hong Kong		OECD		Difference in Scores (HK - OECD)
	Score	S.E.	Score	S.E.	
5th	390	(5.1)	343	(0.9)	47 ***
10th	428	(4.9)	376	(0.7)	52 ***
25th	492	(3.5)	433	(0.6)	59 ***
50th	559	(3.0)	497	(0.6)	62 ***
75th	622	(3.1)	560	(0.6)	62 ***
90th	673	(3.9)	613	(0.7)	60 ***
95th	703	(4.7)	643	(0.8)	60 ***

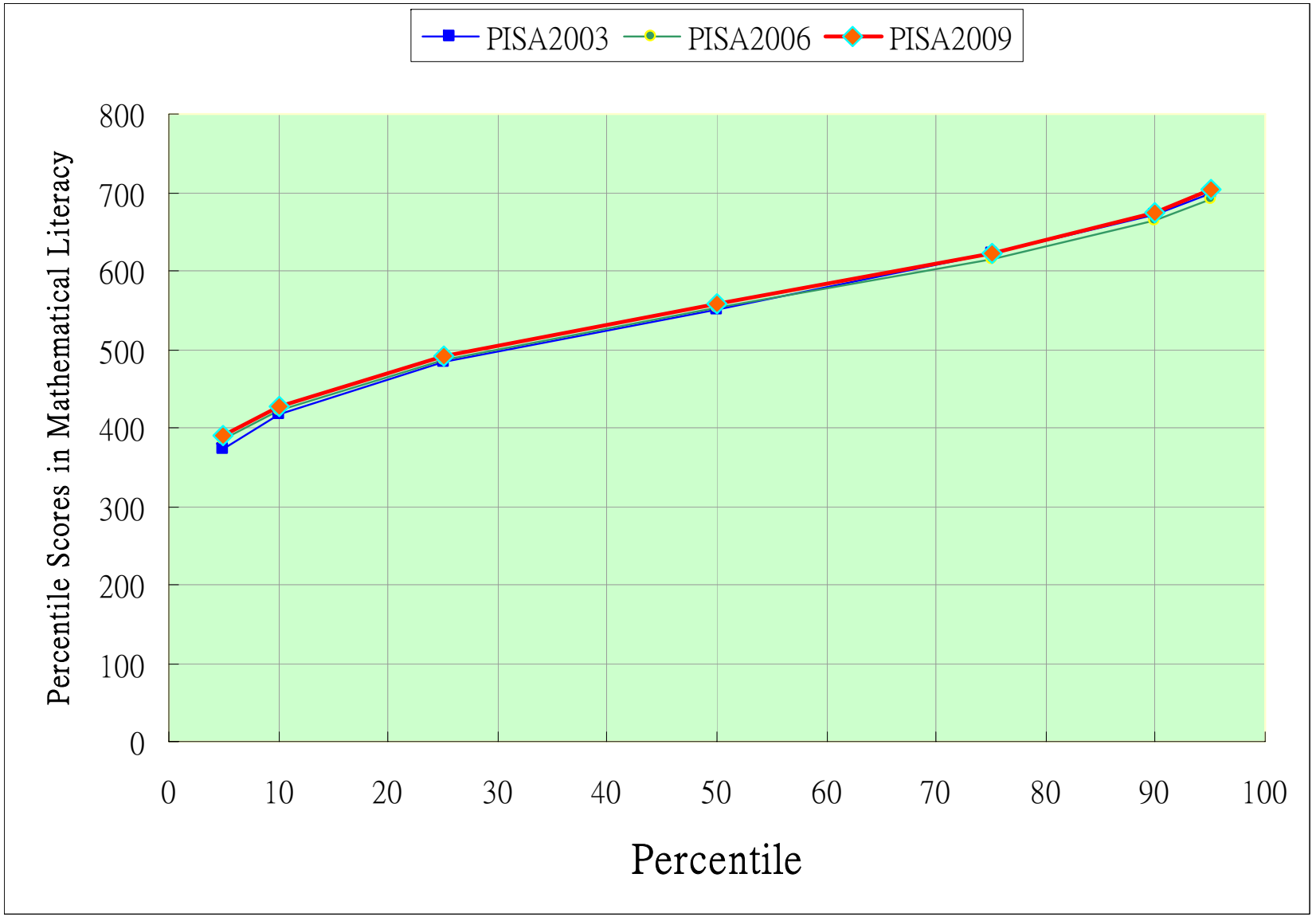
*** Mean difference is significant at the 0.001 level.

Comparison of Percentile Scores between Hong Kong and OECD Average in Mathematical Literacy

at Different Percentiles



Percentile Scores in Mathematical Literacy from 2003 to 2006, and to 2009



Comparison of the Percentage of Correct Answers (1)
HKPISA 2003 through HKPISA 2009
(on the 35 common Mathematics items)

Distribution of Items	Number of items	Average Percent Correct			
		2009	2006	2003	Range of Variation (percentage points)
by Curricular Strands (Contents)					
Algebra	1	22.6	21.7	18.9	3.7
Discrete Mathematics	2	62.3	59.2	60.3	3.1
Functions	2	52.7	50.4	48.3	4.4
Geometry	8	53.1	52.5	53.6	1.1
Number	11	66.6	64.8	65.0	1.8
Probability	2	68.4	70.2	65.6	4.6
Statistics	9	57.9	56.9	55.8	2.1
by "Overarching Ideas"					
Change and Relationships	9	55.8	55.1	53.6	2.2
Quantity	11	64.9	63.2	63.4	1.7
Space and Shape	8	53.1	52.5	53.6	1.1
Uncertainty	7	61.0	59.4	57.8	3.2

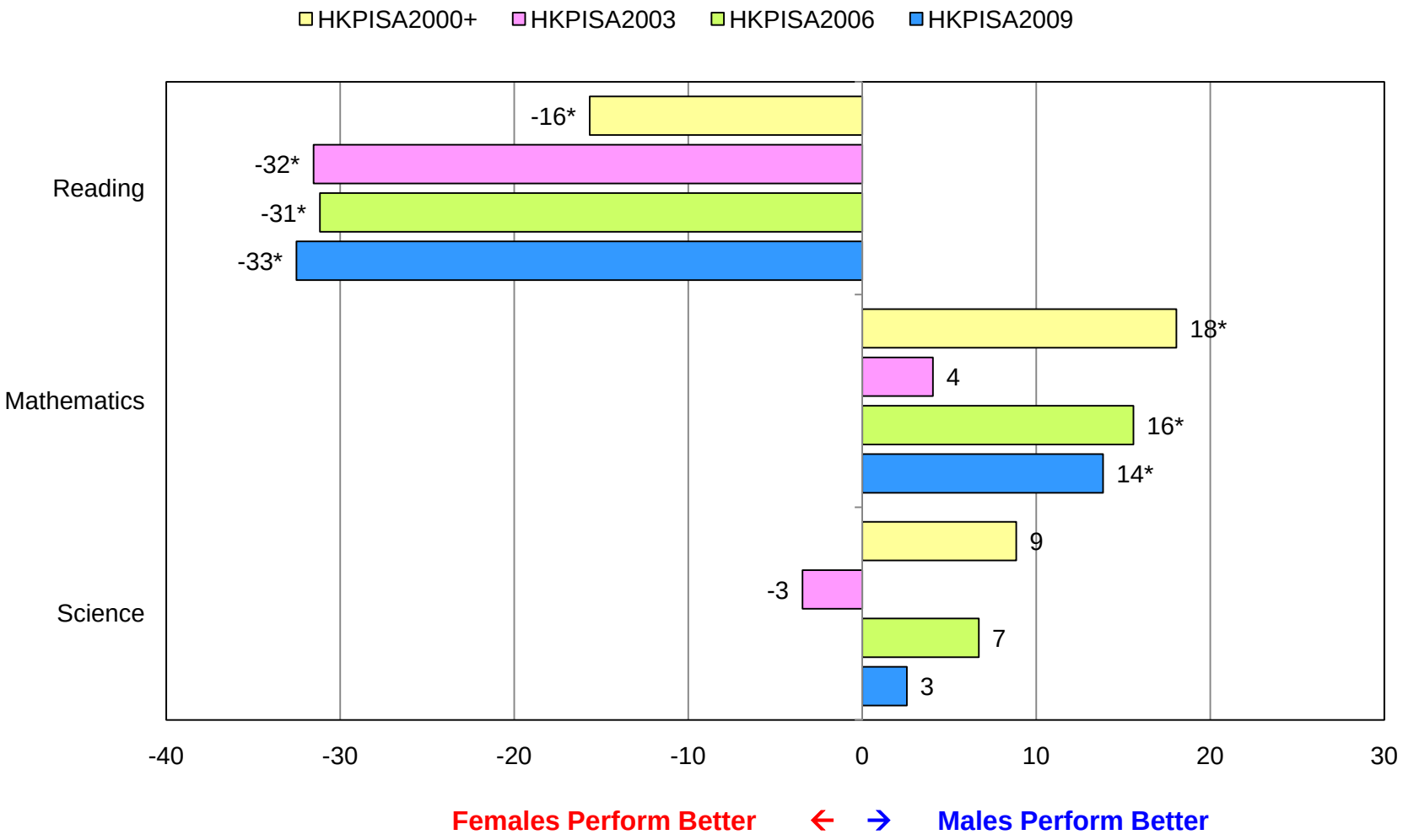
Comparison of the Percentage of Correct Answers (2)
HKPISA 2003 through HKPISA 2009
(on the 35 common Mathematics items)

Distribution of Items	Number of items	Average Percent Correct			Range of Variation (percentage points)
		2009	2006	2003	
by Competency Clusters (Processes)					
Reproduction	9	73.7	72.1	72.2	1.6
Connections	18	56.7	56.0	55.4	1.3
Reflection	8	48.1	46.1	45.8	2.3

The same pattern of ***declining performance*** when progressing ***from reproduction, to connections and to reflection*** is observed in all the three PISA studies.

Gender Differences in Scientific, Reading & Mathematical Literacy

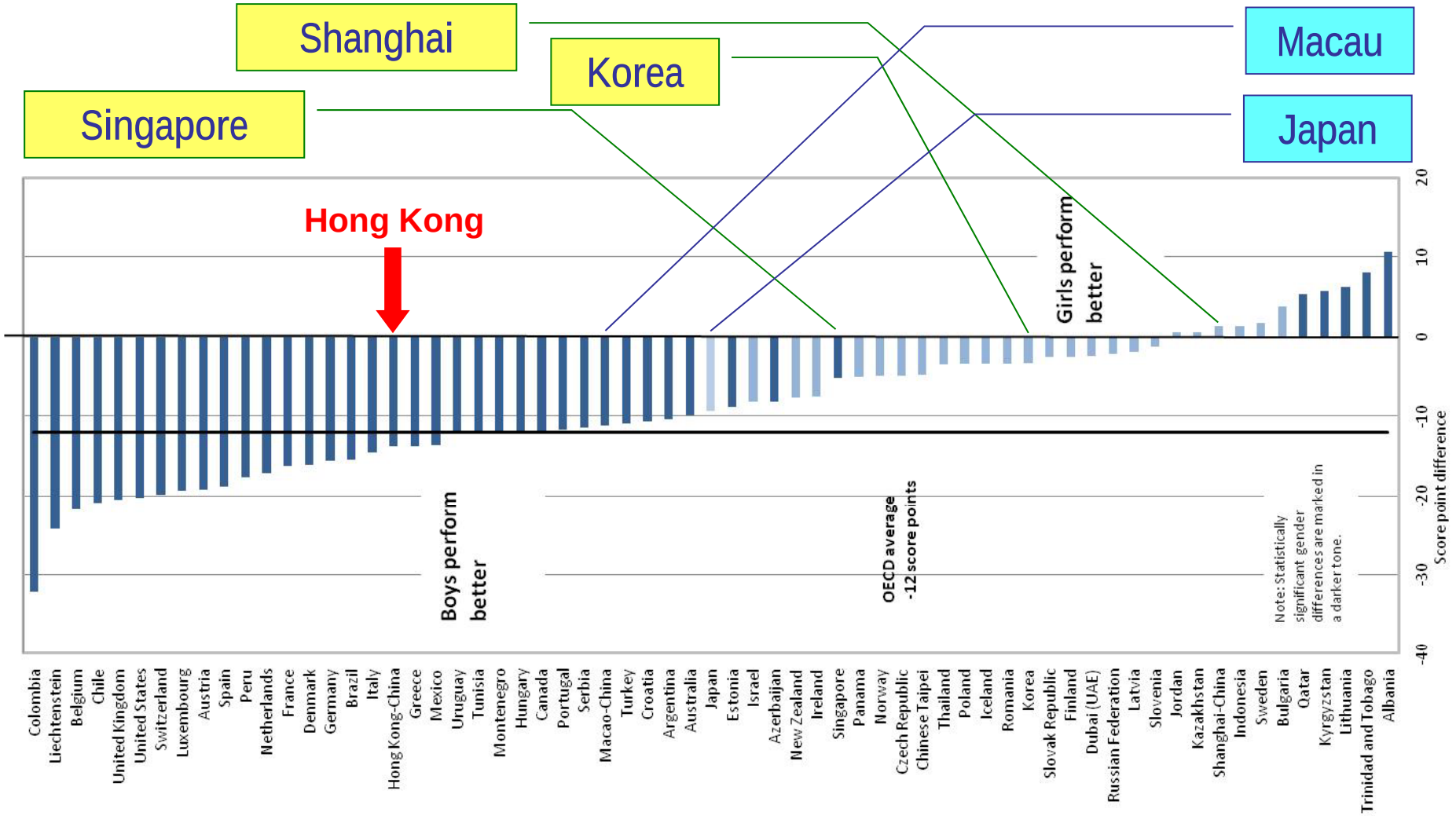
in HKPISA 2000+, HKPISA 2003, HKPISA 2006 and HKPISA 2009



Note:

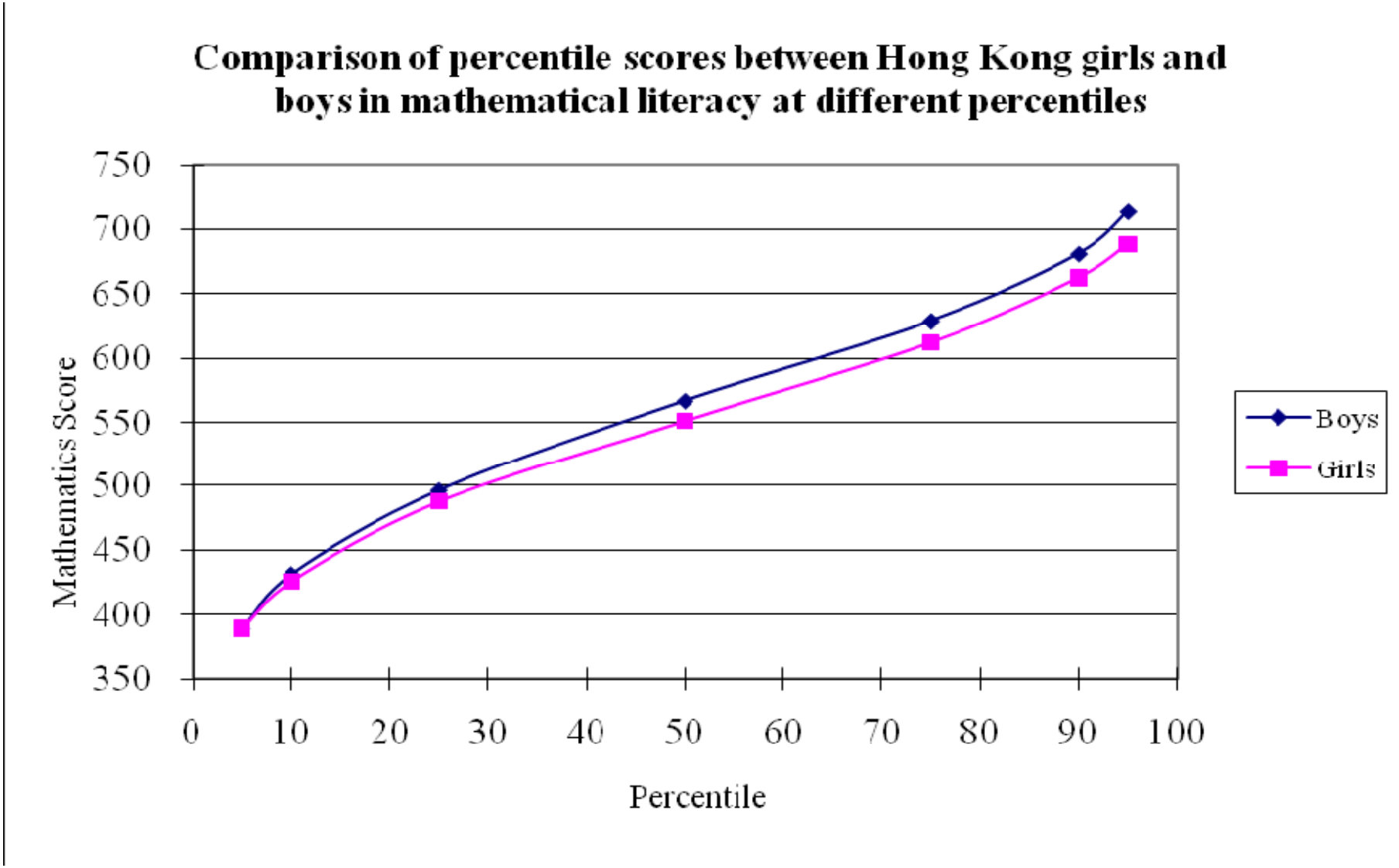
1. Values that are statistically significant are indicated by an asterisk *.
2. This graph is reproducing Figure 5.6.1 from Preliminary Report (p.24).

Gender Differences in Mathematical Literacy in PISA 2009



Boys are better than Girls (1)

Percentile Scores on the scale of mathematical literacy



Boys are better than Girls (1)

Percentile Scores on the scale of mathematical literacy

Percentile Scores of Hong Kong Girls and Boys

Percentile	Boys		Girls		Differences	
	Score	S.E.	Score	S.E.	(Boys - Girls)	
5th	389	(9.3)	389	(6.2)	0	
10th	431	(7.3)	425	(6.0)	6	
25th	496	(5.2)	488	(4.9)	9	
50th	567	(4.9)	551	(3.5)	16	**
75th	629	(4.2)	612	(4.0)	17	**
90th	681	(5.6)	663	(4.7)	19	*
95th	714	(6.0)	689	(4.3)	25	***
Whole Population	561	(4.2)	547	(3.4)	14	*

* Score difference is significant at the 0.05 level. ** Score difference is significant at the 0.01 level. *** Score difference is significant at the 0.001 level.

Boys are better than Girls (2)

at different Proficiency Levels of mathematical literacy

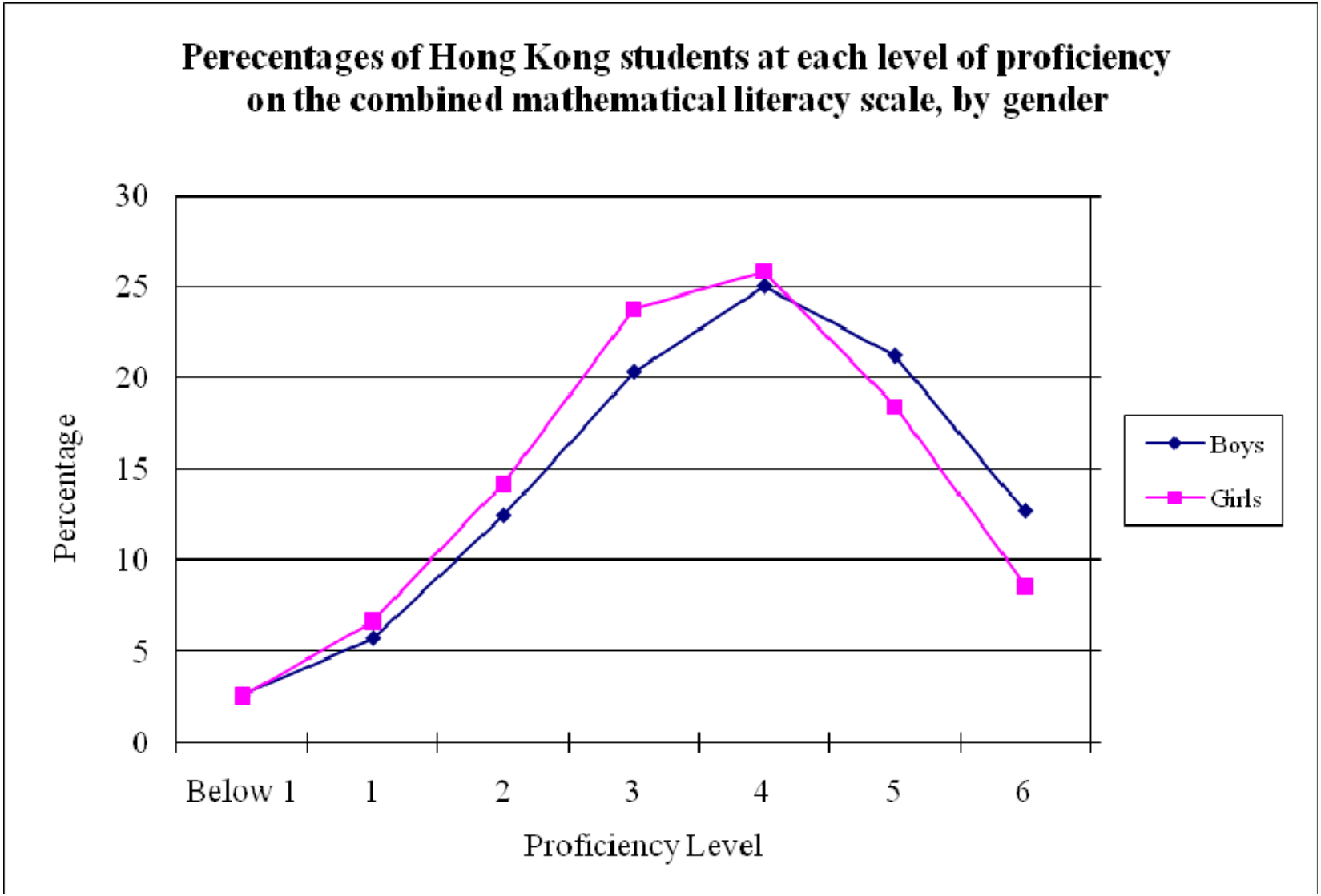
Proportion of HK students at each level of proficiency by gender

Proficiency Level	Boys		Girls		Difference in Percentage Points (Boys - Girls)
	%	S.E.	%	S.E.	
6	12.7	(1.3)	8.6	(0.9)	4.1 **
5	21.2	(1.2)	18.4	(1.1)	2.8
4	25.0	(1.1)	25.8	(1.2)	-0.8
3	20.3	(1.2)	23.8	(1.2)	-3.4 *
2	12.4	(1.2)	14.2	(1.0)	-1.8
1	5.7	(0.8)	6.7	(0.7)	-0.9
Below 1	2.6	(0.6)	2.5	(0.5)	0.1

* Difference is significant at the 0.05 level. ** Difference is significant at the 0.01 level.

Boys are better than Girls (2)

at different Proficiency Levels of mathematical literacy



Conclusion

- Not be concerned too much with ranking.
- Performance in mathematical area still strong – much better than most other countries.
- Performance **stable** and consistently gratifying throughout the years (2003 to 2009).
- With such good grounds, we may target at preparing our students in their "**mathematical literacy**" in its more general sense **adaptable to the technological advanced world in wide-ranging contexts**, not only those calling for reproduction of mathematical skills.
- **gender difference** higher than desirable, especially among high-achievers. call for more attention in mathematics teaching.