Hong Kong Students' Performance in Mathematical Literacy

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Mathematical Literacy in PISA

Definition and its distinctive features

"an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals in recognising the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens."

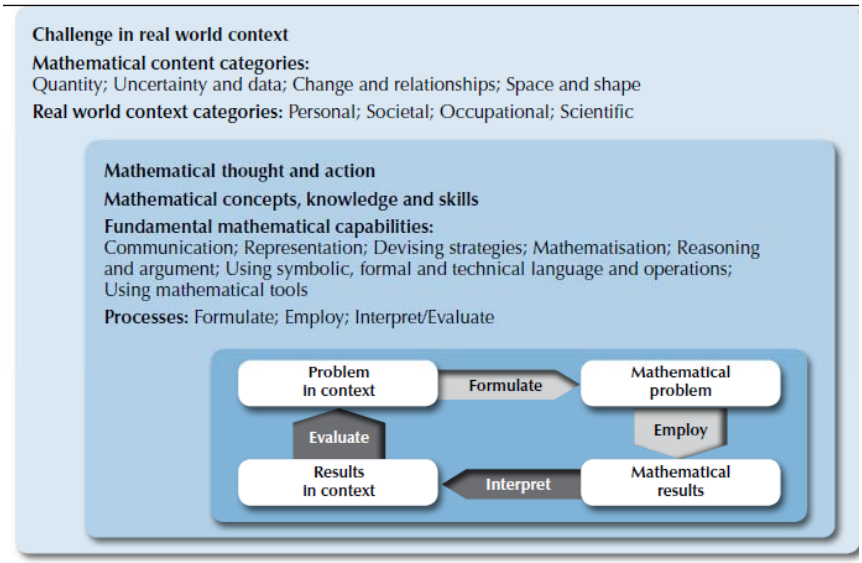
(OECD, 2013, p.25)

Mathematical Literacy in PISA

Mathematical literacy is related to wider, functional use of mathematics. ***Engagement with mathematics*** includes the ability to recognise and formulate mathematical problems in various situations.	
Knowledge Domain (Content)	Clusters of relevant mathematical areas and concepts: • ***Quantity*** • ***Space and shape*** • ***Change and relationships*** • ***Uncertainty and data***
Processes	• ***Formulate*** • ***Employ*** • ***Interpret/Evaluate***
Context and situation	Various areas of application of mathematics, focusing on uses in different settings: • ***Personal*** • ***Societal*** • ***Occupational*** • ***Scientific***

Mathematical Literacy in PISA

Main features of the PISA 2012 mathematics framework



OECD (2013, p.26)

Performance in Mathematical Literacy
of Participating Countries/Regions in PISA 2012

Country/Region	Mean	S.E.	Significance
Shanghai-China	613	(3.3)	▲
Singapore	573	(1.3)	▲
Hong Kong-China	561	(3.2)	--
Chinese Taipei	560	(3.3)	O
Korea	554	(4.6)	O
Macao-China	538	(1.0)	▼
Japan	536	(3.6)	▼
Liechtenstein	535	(4.0)	▼
Switzerland	531	(3.0)	▼
Netherlands	523	(3.5)	▼
Estonia	521	(2.0)	▼
Finland	519	(1.9)	▼
…	…	…	
OECD Average	494	(0.5)	▼

Remarks
▲ denotes score that is significantly higher than that of Hong Kong
O denotes score that is not significantly different from that of Hong Kong
▼ denotes score that is significantly lower than that of Hong Kong

Performance in Mathematical Literacy
of Participating Countries/Regions in PISA 2012

Country/Region	Mean	S.E.	Significance
OECD Average	494	(0.5)	▼
…	…	…	
Uruguay	409	(2.8)	▼
Costa Rica	407	(3.0)	▼
Albania	394	(2.0)	▼
Brazil	391	(2.1)	▼
Argentina	388	(3.5)	▼
Tunisia	388	(3.9)	▼
Jordan	386	(3.1)	▼
Colombia	376	(2.9)	▼
Qatar	376	(0.8)	▼
Indonesia	375	(4.0)	▼
Peru	368	(3.7)	▼

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Hong Kong Students' Performance in Mathematics, Science and Reading from PISA 2000+ to 2012

	Mathematics		Science		Reading	
Cycle	Mean	S.E.	Mean	S.E.	Mean	S.E.
2000+	560	3.3	(541)	3.0	(525)	2.9
2003	550	4.5	(539)	4.3	(510)	3.7
2006	(547)	2.7	(542)	2.5	(536)	2.4
2009	555	2.7	549	2.8	(533)	2.1
2012	561	3.2	555	2.6	545	2.8

* Values in parentheses are significantly different from the mean scores of PISA 2012.

Ranks and Mean Scores in Mathematical Literacy of Top Ranking Countries in the four Cycles of PISA

Country/ Region	PISA 2012 Rank (mean score)	PISA 2009 Rank (mean score)	PISA 2006 Rank (mean score)	PISA 2003 Rank (mean score)
Shanghai-China	1 (613)	1 (600)	/	/
Singapore	2 (573)	2 (562)	/	/
Hong Kong-China	3 (561)	3 (555)	3 (547)	1 (550)
Chinese Taipei	4 (560)	5 (543)	1 (549)	/
Korea	5 (554)	4 (546)	4 (547)	3 (542)
Macao-China	6 (538)	12 (525)	8 (525)	9 (527)
Japan	7 (536)	9 (529)	10 (523)	6 (534)
Liechtenstein	8 (535)	7 (536)	9 (525)	5 (536)
Switzerland	9 (531)	8 (534)	6 (530)	10 (527)
Netherlands	10 (523)	11 (526)	5 (531)	4 (538)

Proficiency level

Mathematical Proficiency Levels

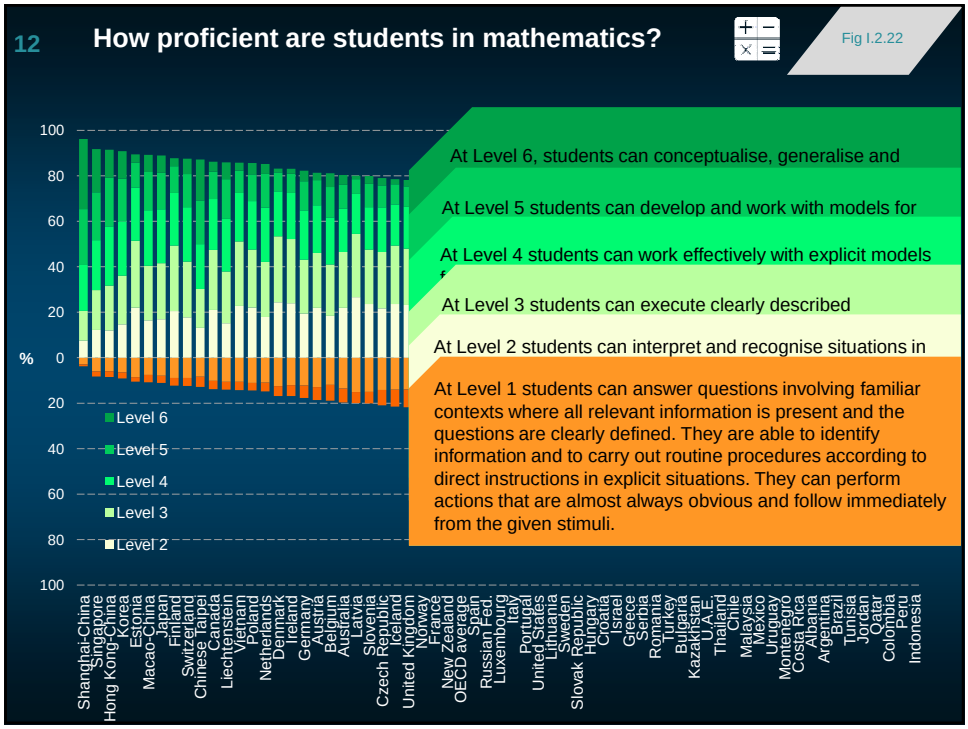
Score Range of the Mathematical Proficiency Levels

Proficiency Levels	Lower Score Limit
6	669.3
5	607.0
4	544.7
3	482.4
2	420.1
1	357.8
Below 1	Below 357.8

Proficiency Levels 1 – 6

- General ability of an individual in mathematics and related areas, and thus his/her ***prospects and capacity to participate fully in the society***
- Also implications for the role that the country will play in the advancing technological world, i.e. ***the country's competitiveness***

Level	Lower score limit	What students can typically do at each level
6	669.3	At Level 6 students can conceptualise, generalise and utilise information based on their investigations and modelling of complex problem situations. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply their insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments and the appropriateness of these to the original situations.
5	607.0	At Level 5 students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.
4	544.7	At Level 4 students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilise well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.
3	482.4	At Level 3 students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications when reporting their interpretations, results and reasoning.
2	420.1	At Level 2 students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.
1	357.8	At Level 1 students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and

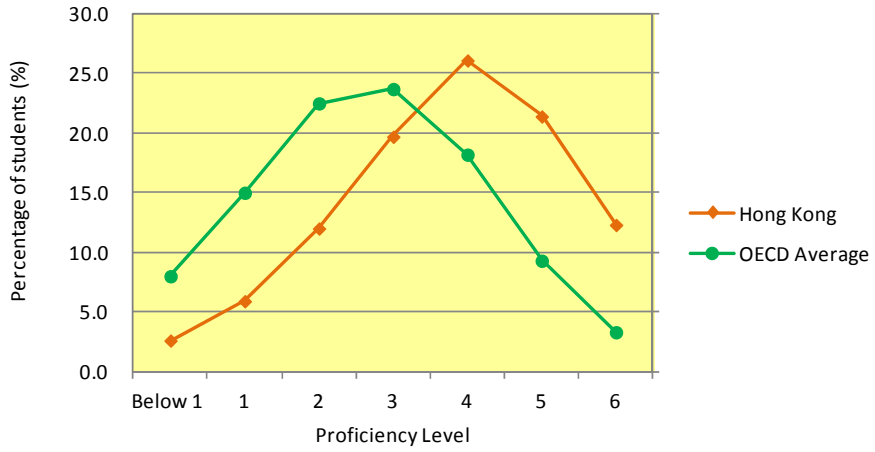


Percentage of Students
at each Level of Proficiency
on the scale of mathematical literacy in PISA 2012
Hong Kong vs OECD Average

	Hong Kong	OECD Average	Difference (HK – OECD)
Level 6	12.3%	3.3%	**+9.0% *****
Level 5	21.4%	9.3%	**+12.1% *****
Level 4	26.1%	18.2%	**+7.9% *****
Level 3	19.7%	23.7%	-4.0% ***
Level 2	12.0%	22.5%	-10.4% ***
Level 1	5.9%	15.0%	-9.0% ***
Below Level 1	2.6%	8.0%	-5.4% ***

*** Difference is significant at 0.001 level.

Percentage of Students
at each Level of Proficiency
on the scale of mathematical literacy in PISA 2012
Hong Kong vs OECD Average



Percentage of Students at Level 2, 3, 4, 5 & 6 on the Overall Mathematical Literacy Scale of the Top 10 Countries/Regions

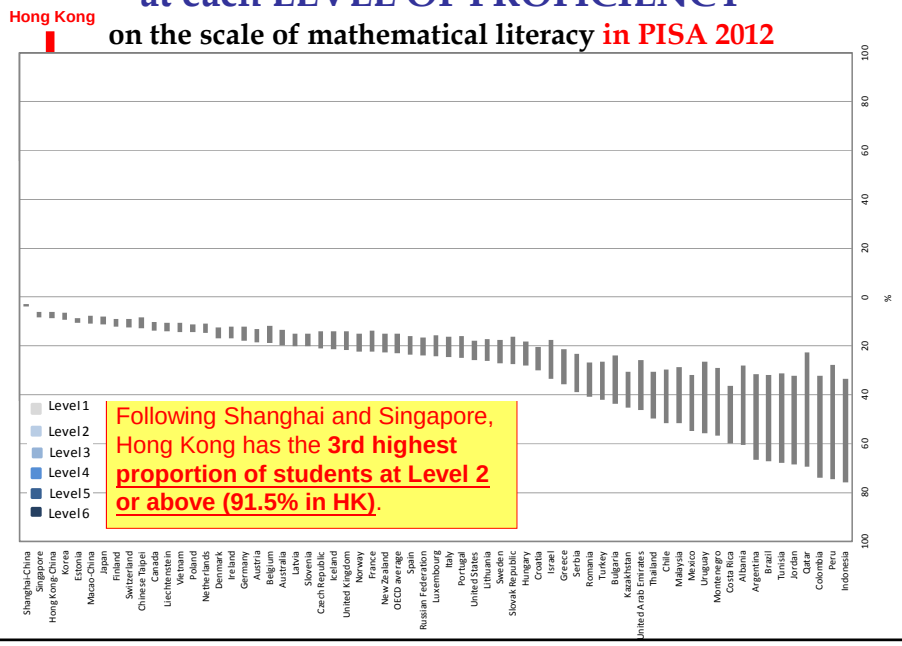
(PISA 2012)

Country/Region	Mean Score	Level 2 (%)	Level 3 (%)	Level 4 (%)	Level 5 (%)	Level 6 (%)
Shanghai-China	613	7.5	13.1	20.2	24.6	30.8
Singapore	573	12.2	17.5	22.0	21.0	19.0
Hong Kong-China	561	12.0	19.7	26.1	21.4	12.3
Chinese Taipei	**560**	13.1	17.1	19.7	19.2	18.0
Korea	554	14.7	21.4	23.9	18.8	12.1
Macao-China	538	16.4	24.0	24.4	16.8	7.6
Japan	536	16.9	24.7	23.7	16.0	7.6
Liechtenstein	535	15.2	22.7	23.2	17.4	7.4
Switzerland	531	17.8	24.5	23.9	14.6	6.8
Netherlands	523	17.9	24.2	23.8	14.9	4.4

Percentage of students

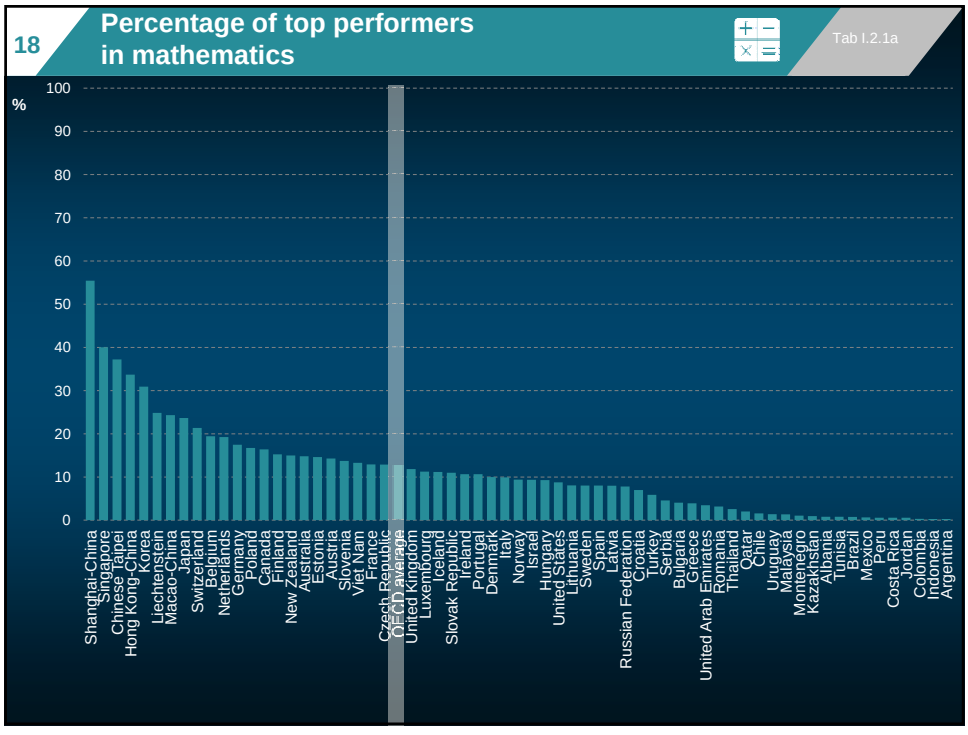
at each LEVEL OF PROFICIENCY

on the scale of mathematical literacy in PISA 2012



Percentage of Students at Proficiency Level 5 or Above in Countries/Regions with a Total of More Than 20% in PISA 2012

Country/Region	Percentage at Level 5 (606.99 – 669.30)	Percentage at Level 6 (above 669.30)	Total Percentage at Level 5 or Above
Shanghai-China	24.6%	30.8%	55.4%
Singapore	21.0%	19.0%	40.0%
Chinese Taipei	19.2%	18.0%	37.2%
Hong Kong	**21.4%**	**12.3%**	**33.7%**
Korea	18.8%	12.1%	30.9%
Liechtenstein	17.4%	7.4%	24.8%
Macao-China	16.8%	7.6%	24.3%
Japan	16.0%	7.6%	23.7%
Switzerland	14.6%	6.8%	21.4%
OECD countries	*9.3%*	*3.3%*	*12.6%*



Percentage of Hong Kong Students at each Level of Proficiency on the scale of mathematical literacy (2003 to 2012)

	PISA 2003	PISA 2006		PISA 2009		PISA 2012	
Level 6	10.5	9.0	(–1.5)	10.8	(+1.8)	**12.3**	(+1.5)
Level 5	20.2	18.7	(–1.4)	19.9	(+1.2)	**21.4**	(+1.5)
Level 4	25.0	25.6	(+0.6)	25.4	(–0.2)	**26.1**	(+0.7)
Level 3	20.0	22.7	(+2.8)	21.9	(–0.8)	**19.7**	(-2.3)
Level 2	13.9	14.4	(+0.5)	13.2	(–1.2)	**12.0**	(-1.2)
Level 1	6.5	6.6	(+0.1)	6.2	(–0.4)	**5.9**	(-0.2)
Below Level 1	3.9	2.9	(–1.0)	2.6	(–0.4)	**2.6**	(0.0)

Numbers in brackets are DIFFERENCES (expressed by percentage points) from the corresponding percentages in the *previous* PISA cycle.

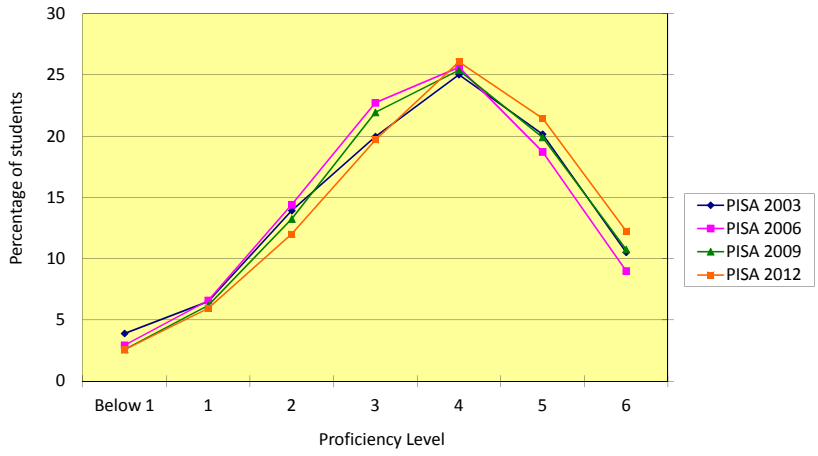
The differences at all Levels of Proficiency between two successive years are **statistically insignificant**.

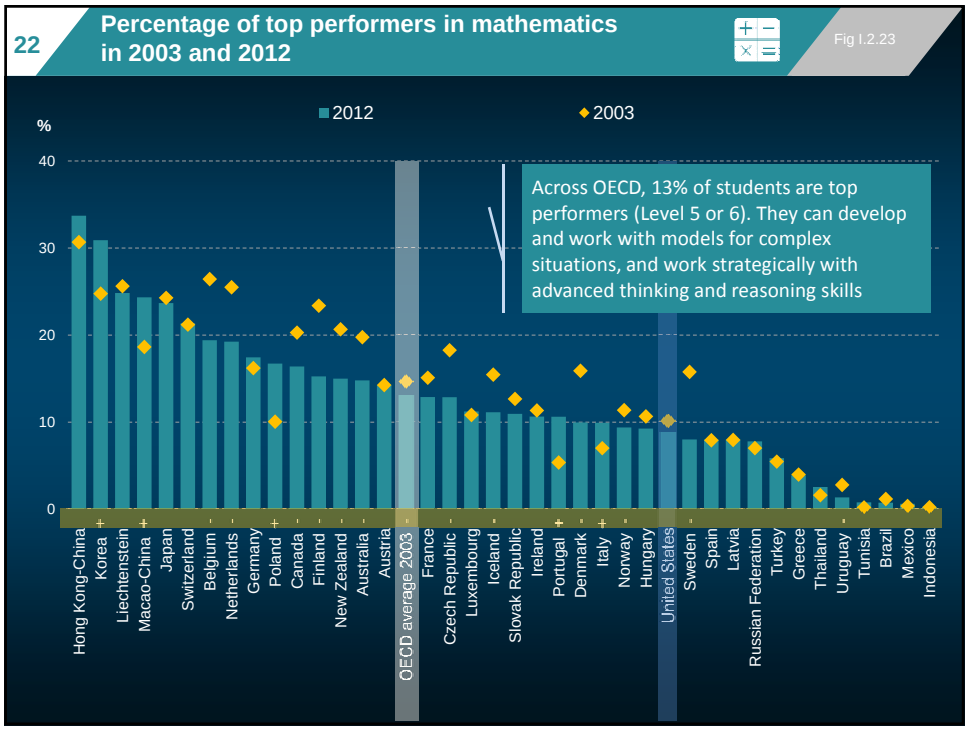
Percentages of Hong Kong Students at Each Level of Proficiency on the Mathematical Literacy Scale in PISA 2003, 2006, 2009 and 2012

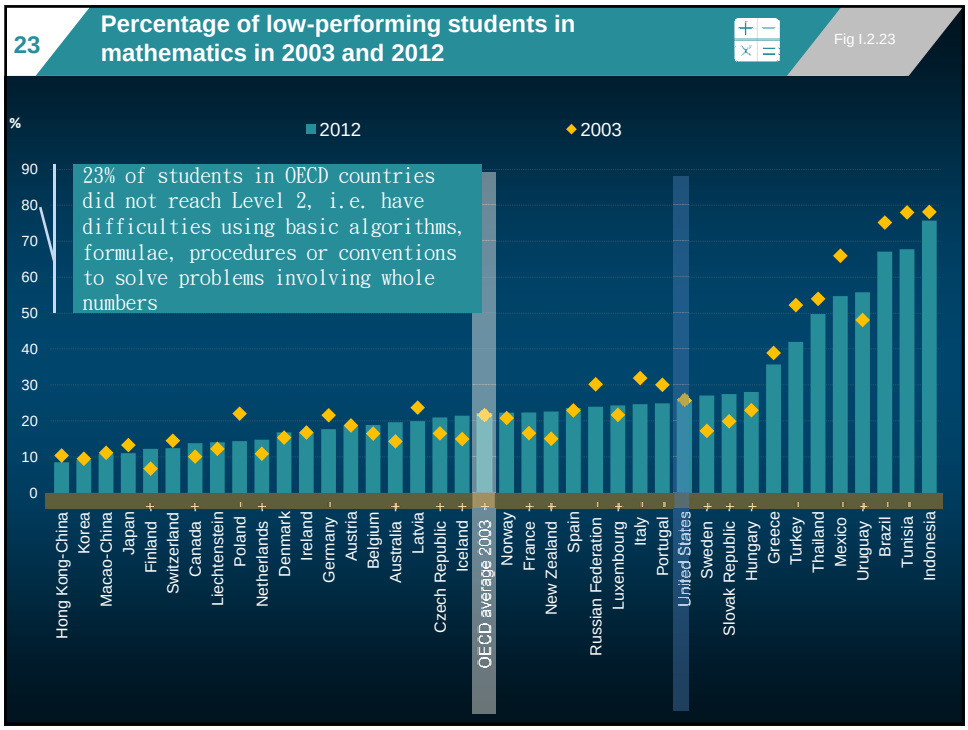
Proficiency Level	PISA 2003		PISA 2006		PISA 2009		PISA 2012	
	%	S.E.	%	S.E.	%	S.E.	%	S.E.
6	10.5	(0.9)	9.0	(0.8)	10.8	(0.8)	12.3	(0.9)
5	20.2	(1.0)	18.7	(0.8)	19.9	(0.8)	21.4	(1.0)
4	25.0	(1.2)	25.6	(0.9)	25.4	(0.9)	26.1	(1.1)
3	20.0	(1.2)	22.7	(1.1)	21.9	(0.8)	19.7	(1.0)
2	13.9	(1.0)	14.4	(0.8)	13.2	(0.7)	12.0	(0.8)
1	6.5	(0.6)	6.6	(0.6)	6.2	(0.5)	5.9	(0.6)
Below 1	3.9	(0.7)	2.9	(0.5)	2.6	(0.4)	2.6	(0.4)

Percentage of Students
at each Level of Proficiency
on the scale of mathematical literacy in PISA in Hong Kong

from 2003 to 2012







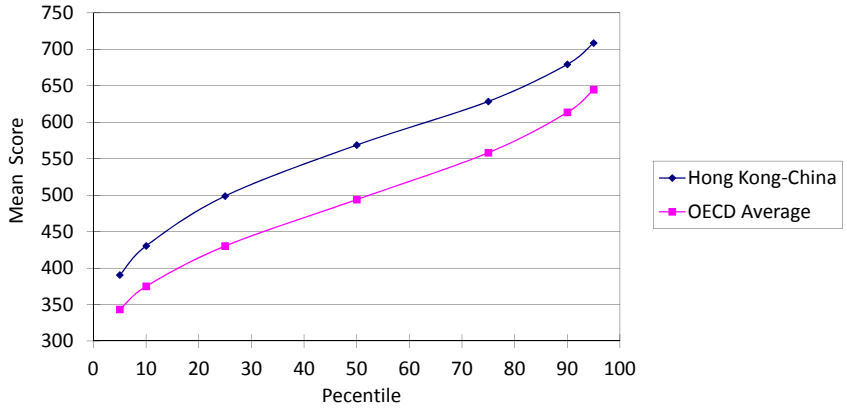
Comparison of Scores between Hong Kong and OECD Average in Mathematical Literacy

at Different Percentiles in PISA 2012

Percentile	Hong Kong		OECD		Difference in Scores (HK - OECD)
	Score	S.E.	Score	S.E.	
5th	391	(5.9)	343	(0.8)	47 ***
10th	430	(6.2)	375	(0.7)	55 ***
25th	499	(4.7)	430	(0.6)	69 ***
50th	569	(3.8)	494	(0.6)	75 ***
75th	629	(3.5)	558	(0.6)	70 ***
90th	679	(4.2)	614	(0.7)	66 ***
95th	709	(4.3)	645	(0.8)	64 ***

*** Mean difference is significant at the 0.001 level.

Comparison of Scores between Hong Kong and OECD Average in Mathematical Literacy at Different Percentiles in PISA 2012



Comparison of Hong Kong's Percentile Scores in Mathematical Literacy at Different Percentiles in the Five Cycles of PISA

Percentile	PISA 2000+		PISA 2003		PISA 2006		PISA 2009		PISA 2012	
	Score	S.E.	Score	S.E.	Score	S.E.	Score	S.E.	Score	S.E.
5th	390	(10.3)	374	(11.0)	386	(6.1)	390	(5.1)	391	(5.9)
10th	434	(7.6)	417	(8.0)	423	(6.4)	428	(4.9)	430	(6.2)
25th	502	(4.5)	485	(6.9)	486	(4.5)	492	(3.5)	499	(4.7)
50th	570	(3.8)	559	(4.8)	552	(2.7)	559	(3.0)	569	(3.8)
75th	626	(3.9)	622	(3.7)	614	(3.1)	622	(3.1)	629	(3.5)
90th	673	(5.1)	672	(4.1)	665	(3.5)	673	(3.9)	679	(4.2)
95th	699	(5.0)	700	(4.0)	692	(4.8)	703	(4.7)	709	(4.3)

Percentile	Difference			
	2012-2000+	2012-2003	2012-2006	2012-2009
5th	1	17	5	1
10th	-4	13	7	2
25th	-3	14	13	6
50th	-1	9	17 ***	10 *
75th	2	7	14 **	7
90th	6	8	14 **	7
95th	10	9	17 **	6

Subscales

Percentage of Correct Answers (1)

Hong Kong and the OECD Average

Distribution of Items	Number of items	Percent Correct Hong Kong	Percent Correct OECD Average
by Contents			
Change and Relationships	21	**56**	41
Quantity	21	**73**	59
Space and Shape	21	**53**	38
Uncertainty and Data	21	**64**	52
by Processes			
Employ	36	**64**	49
Formulate	28	**51**	36
Interpret	20	**72**	61

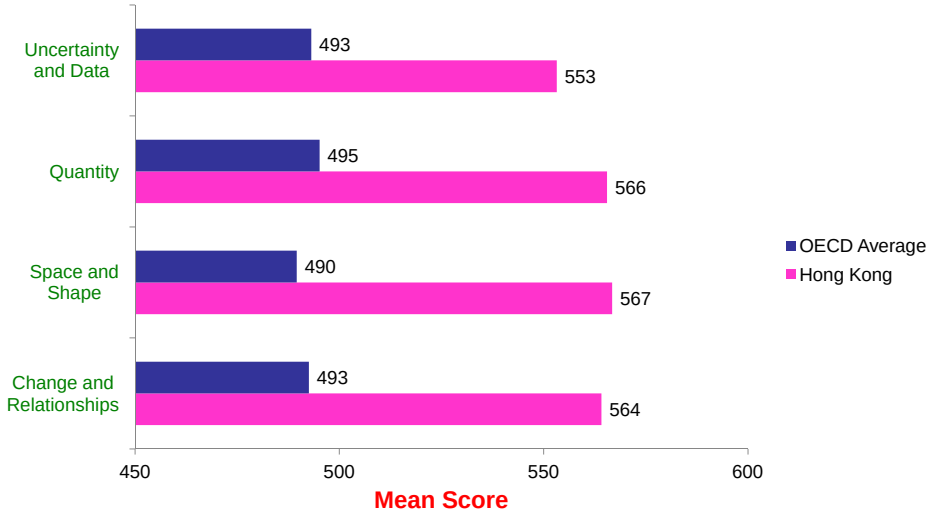
By Contents and Processes, the **percentage of correct answers** of Hong Kong 15-year-old students is **HIGHER** than that of the OECD Average.

Comparison of the Percentage of Correct Answers
HKPISA 2003 through HKPISA 2012
(on the 34 common Mathematics items)

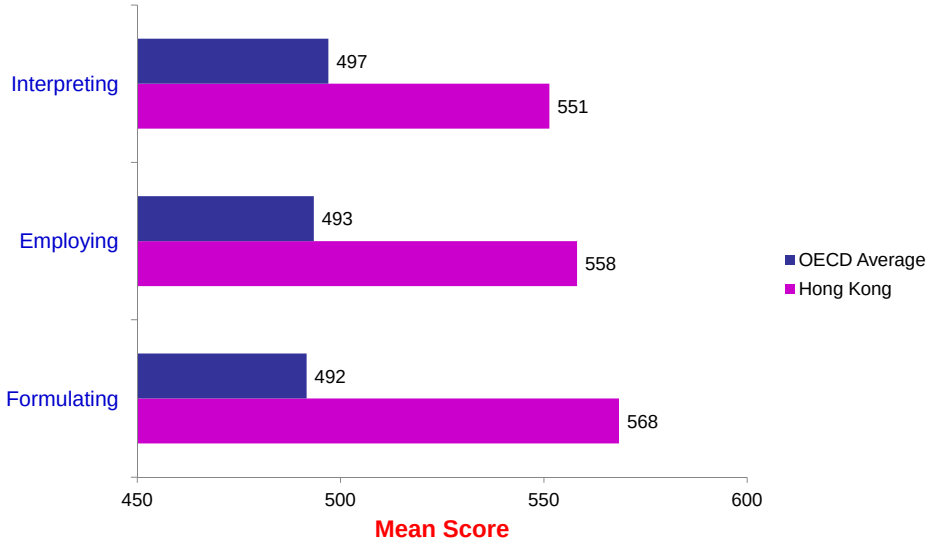
Distribution of Items	Number of items	Average Percent Correct				
		2012	2009	2006	2003	Range of Variation (percentage points)
by Contents						
Change and Relationships	9	56.7	55.8	55.1	53.6	3.1
Quantity	10	69.3	66.4	65.0	64.9	4.5
Space and Shape	8	53.5	53.1	52.5	53.6	1.1
Uncertainty and Data	7	62.0	61.0	59.4	57.8	4.2
by Processes						
Formulate	10	52.3	52.3	50.9	49.6	2.7
Employ	14	61.4	59.2	57.9	58.6	3.5
Interpret	10	68.3	66.7	66.1	64.8	3.5

The same pattern of ***declining performance*** when progressing ***from "Interpret", to "Employ" and to "Formulate"*** is observed in all the four PISA studies.

Performance on the Sub-scale of Contents



Performance on the Sub-scale of Processes



Percentage of Students at Each Level of Mathematical Proficiency by Process

Proficiency Level	Employ			Formulate			Interpret		
	HK	OECD	Diff.	HK	OECD	Diff.	HK	OECD	Diff.
6	9.2%	2.8%	6.3%	19.2%	5.0%	14.2%	9.4%	4.2%	5.1%
5	21.9%	9.3%	12.6%	19.9%	9.5%	10.4%	19.2%	10.2%	9.0%
4	28.5%	18.6%	9.9%	21.5%	16.6%	4.8%	27.4%	18.5%	8.9%
3	21.0%	24.1%	-3.1%	16.8%	21.6%	-4.8%	21.7%	22.9%	-1.2%
2	11.8%	22.4%	-10.6%	11.9%	21.3%	-9.4%	13.2%	21.1%	-7.9%
1	5.5%	14.6%	-9.1%	6.5%	15.6%	-9.1%	6.4%	14.3%	-7.9%
Below 1	2.0%	8.1%	-6.1%	4.2%	10.3%	-6.2%	2.7%	8.8%	-6.1%

Percentage of Students at Each Level of Mathematical Proficiency by Content

Proficiency Level	Change and Relationships			Quantity			Space and Shape			Uncertainty and Data		
	HK	OECD	Diff.	HK	OECD	Diff.	HK	OECD	Diff.	HK	OECD	Diff.
6	15.0%	4.5%	10.4%	14.6%	3.9%	10.7%	17.1%	4.5%	12.6%	9.2%	3.2%	6.0%
5	21.0%	9.9%	11.1%	22.1%	10.1%	12.1%	20.3%	8.9%	11.4%	20.0%	9.2%	10.7%
4	24.1%	17.5%	6.6%	24.6%	18.5%	6.1%	22.6%	16.3%	6.4%	26.9%	18.1%	8.8%
3	18.8%	22.2%	-3.4%	18.6%	22.9%	-4.3%	18.1%	22.2%	-4.2%	22.5%	23.8%	-1.4%
2	11.9%	20.9%	-9.0%	11.4%	21.1%	-9.7%	12.2%	22.3%	-10.0%	13.2%	22.5%	-9.3%
1	5.9%	14.5%	-8.6%	5.3%	14.3%	-9.0%	6.4%	15.8%	-9.4%	6.0%	14.8%	-8.8%
Below 1	3.3%	10.4%	-7.1%	3.3%	9.2%	-5.9%	3.2%	10.0%	-6.8%	2.3%	8.3%	-6.0%

Comparison of Performance on the Different Process Subscales of Top Ranking Countries

Country performance on the subscale is between 0 to 3 score points **higher** than on the combined mathematics scale.
Country performance on the subscale is between 3 to 10 score points **higher** than on the combined mathematics scale.
Country performance on the subscale is 10 score points **higher** than on the combined mathematics scale.

Country performance on the subscale is between 0 to 3 score points **lower** than on the combined mathematics scale.
Country performance on the subscale is between 3 to 10 score points **lower** than on the combined mathematics scale.
Country performance on the subscale is 10 score points **lower** than on the combined mathematics scale.

	Mathematical score	Performance difference between the combined mathematics scale and each *process* subscale		
		Formulating	*Employing*	*Interpreting*
Shanghai-China	613	12	0	-34
Singapore	573	8	1	-18
Hong Kong-China	561	7	-3	-10
Chinese Taipei	560	19	-11	-11
Korea	554	8	-1	-14
Macao-China	538	7	-2	-9
Japan	536	18	-6	-5
Liechtenstein	535	0	1	5
Switzerland	531	7	-2	-2
Netherlands	523	4	-4	3

Comparison of Performance on the Different Content Subscales of Top Ranking Countries

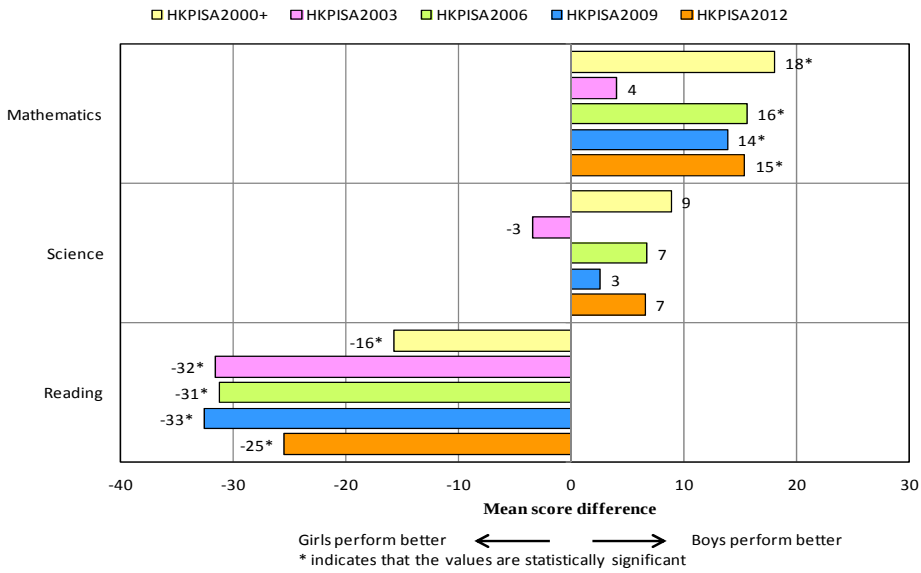
Country performance on the subscale is between 0 to 3 score points **higher** than on the combined mathematics scale.
Country performance on the subscale is between 3 to 10 score points **higher** than on the combined mathematics scale.
Country performance on the subscale is 10 score points **higher** than on the combined mathematics scale.

Country performance on the subscale is between 0 to 3 score points **lower** than on the combined mathematics scale.
Country performance on the subscale is between 3 to 10 score points **lower** than on the combined mathematics scale.
Country performance on the subscale is 10 score points **lower** than on the combined mathematics scale.

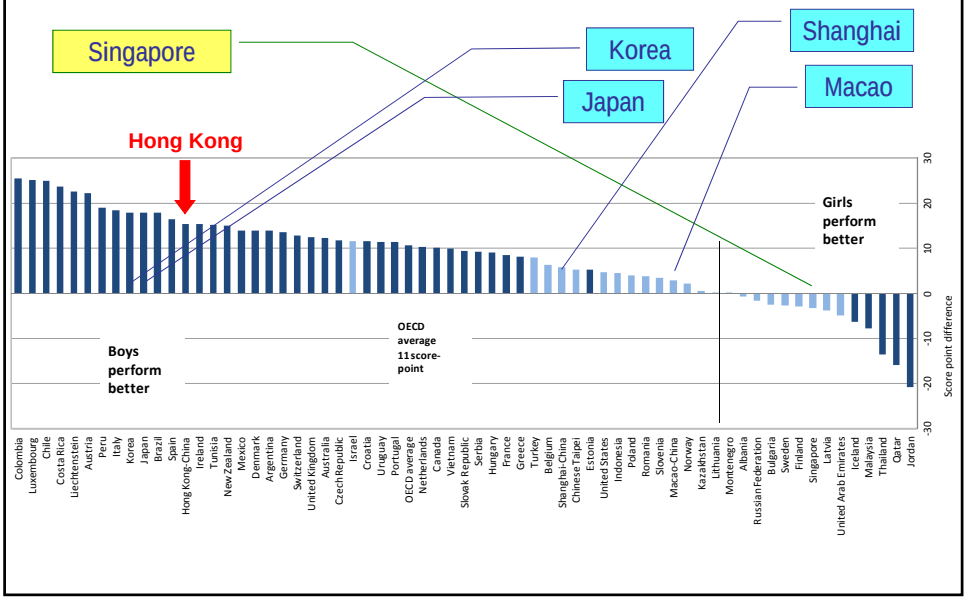
	Mathematical score	Performance difference between the combined mathematics scale and each *content* subscale			
		Change and relationship	*Space and shape*	*Quantity*	*Uncertainty*
Shanghai-China	613	11	36	-22	-21
Singapore	573	7	6	-5	-14
Hong Kong-China	561	3	6	4	-8
Chinese Taipei	560	1	32	-16	-11
Korea	554	5	19	-16	-16
Macao-China	538	4	20	-8	-13
Japan	536	6	21	-18	-8
Liechtenstein	535	7	4	3	-9
Switzerland	531	-1	13	0	-9
Netherlands	523	-5	-16	9	9

Gender difference

Gender Differences in Mathematical, Scientific & Reading Literacy from HKPISA 2000+ to HKPISA 2012



Gender Differences in Mathematical Literacy in PISA 2012



Boys perform better than Girls (1)

Percentile Scores on the scale of mathematical literacy

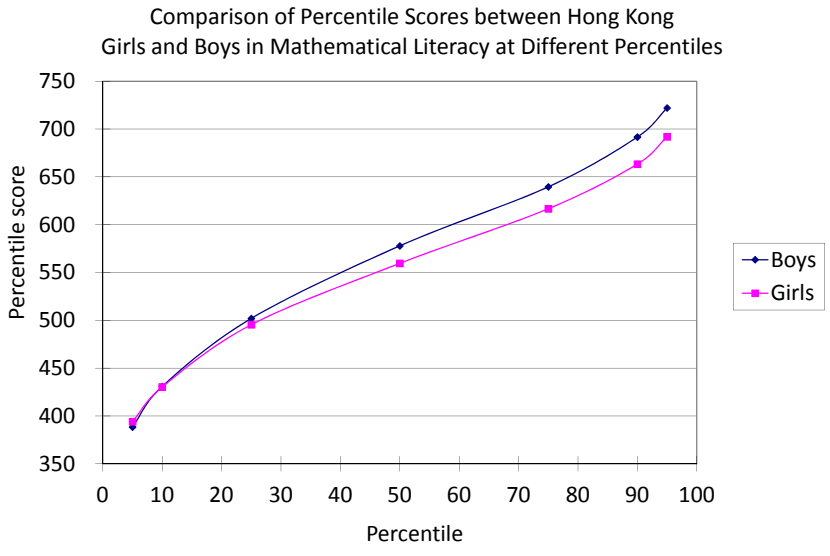
Percentile Scores of Hong Kong Girls and Boys

	Boys		Girls		Differences
Percentile	Score	S.E.	Score	S.E.	(Boys - Girls)
5th	388	(6.0)	394	(6.6)	-6
10th	431	(8.0)	430	(6.8)	0
25th	502	(6.4)	495	(5.0)	6
50th	578	(5.1)	560	(4.2)	18 **
75th	640	(5.2)	617	(4.9)	23 **
90th	692	(5.9)	663	(5.5)	28 ***
95th	722	(6.0)	692	(6.0)	30 ***
Whole Population	568	(4.6)	553	(3.9)	15 **

** Score difference is significant at the 0.01 level. *** Score difference is significant at the 0.001 level.

Boys perform better than Girls (1)

Percentile Scores on the scale of mathematical literacy



Boys are better than Girls (2)

at different Proficiency Levels of mathematical literacy

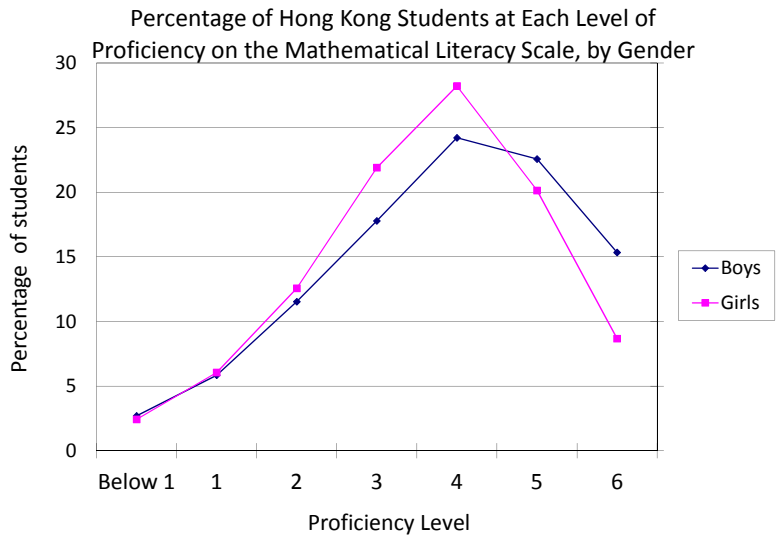
Proportion of HK students at each level of proficiency by gender

Proficiency Level	Boys		Girls		Difference in Percentage Points (Boys - Girls)
	%	S.E.	%	S.E.	
6	15.3	(1.6)	8.7	(1.2)	6.7 ***
5	22.6	(1.5)	20.1	(1.2)	2.4
4	24.2	(1.5)	28.2	(1.5)	-4.0
3	17.8	(1.2)	21.9	(1.6)	-4.1 *
2	11.5	(1.0)	12.6	(1.0)	-1.0
1	5.8	(0.8)	6.1	(0.8)	-0.2
Below 1	2.7	(0.4)	2.4	(0.5)	0.3

* Difference is significant at the 0.05 level. *** Difference is significant at the 0.001 level.

Boys are better than Girls (2)

at different Proficiency Levels of mathematical literacy



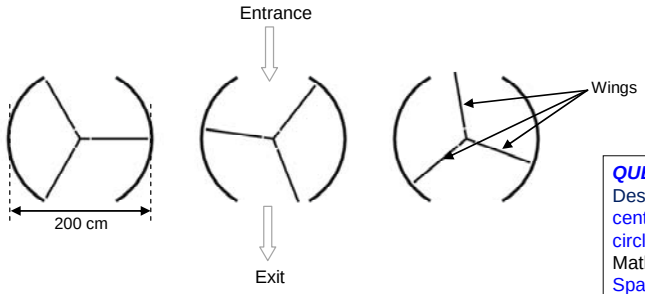
Conclusion

- Not be concerned too much with ranking.
- Performance in mathematical area still strong – much better than most other countries.
- Performance **stable** and consistently gratifying throughout the years (2003 to 2012).
- Performance on the processes of "**formulating**" and "**interpreting**", as well as that on the content area of "**uncertainty and data**", deserve our attention.
- With such good grounds, we may target at preparing our students in their "**mathematical literacy**" in its more general sense **adaptable to the technological advanced world in wide-ranging contexts**, not only those calling for reproduction of mathematical skills.
- **gender difference** higher than desirable, especially among high-achievers; call for more attention in mathematics teaching.

Sample items

REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



QUESTION INTENT:
Description: Compute the central angle of a sector of a circle
Mathematical content area: Space and shape
Context: Scientific
Process: Employ

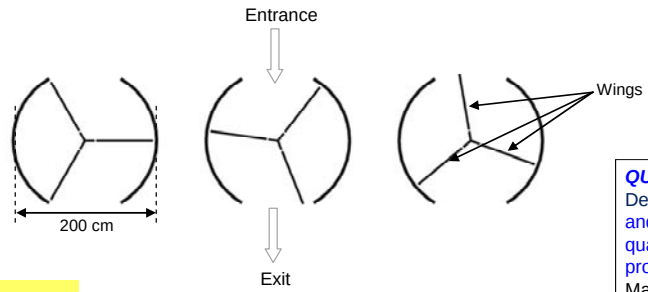
Q1

What is the size in degrees of the angle formed by two door wings?

Full Credit (Level 3)
Code 1: 120 *[accept the equivalent reflex angle: 240]*.

REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



QUESTION INTENT:
Description: Identify information and construct an (implicit) quantitative model to solve the problem
Mathematical content area: Quantity
Context: Scientific
Process: Formulate

Q3

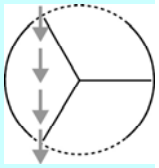
The door makes 4 complete rotations in a minute.
There is room for a maximum of two people in each of the three door sectors.
What is the maximum number of people that can enter the building through the door in 30 minutes?

A. 60 B. 180 C. 240 D. 720

Full Credit (Level 4)
Code 1: D. 720

Q2

The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.



What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: ..... cm

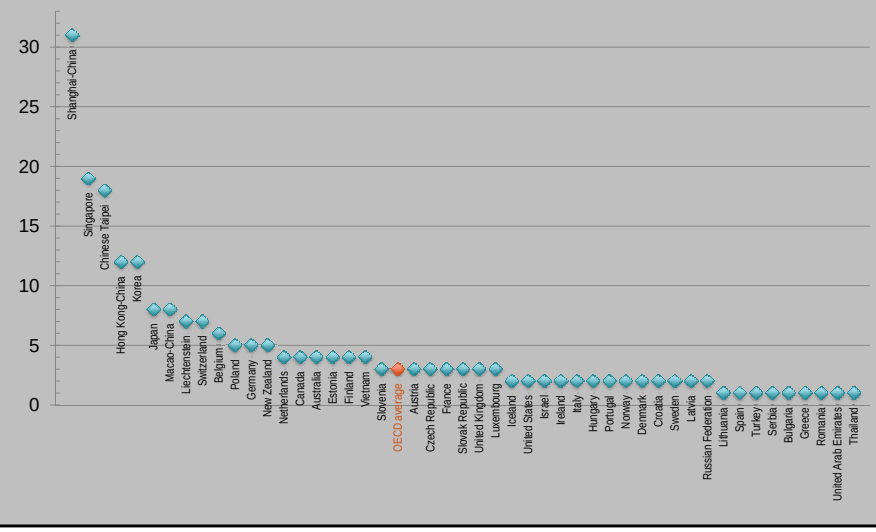
Full Credit ***(Level 6)***

Code 1: Answers in the range from 103 to 105. *[Accept answers calculated as 1/6th of the circumference ($\frac{100\pi}{3}$). Also accept an answer of 100 only if it is clear that this response resulted from using $\pi = 3$. Note: Answer of 100 without supporting working could be obtained by a simple guess that it is the same as the radius (length of a single wing).]*

QUESTION INTENT:
Description: Interpret a geometrical model of a real life situation to calculate the length of an arc
Mathematical content area: Space and shape
Context: Scientific
Process: Formulate

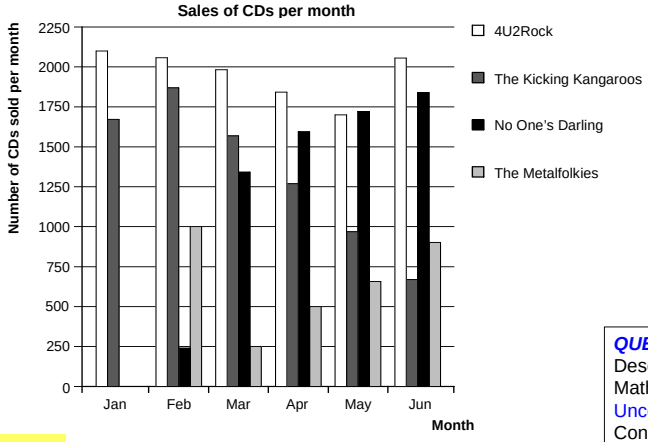
48 PISA 2012 Sample Question – **Revolving Door Q2**

Percent of 15-year-olds who scored Level 6 or Above



CHARTS

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



Q1

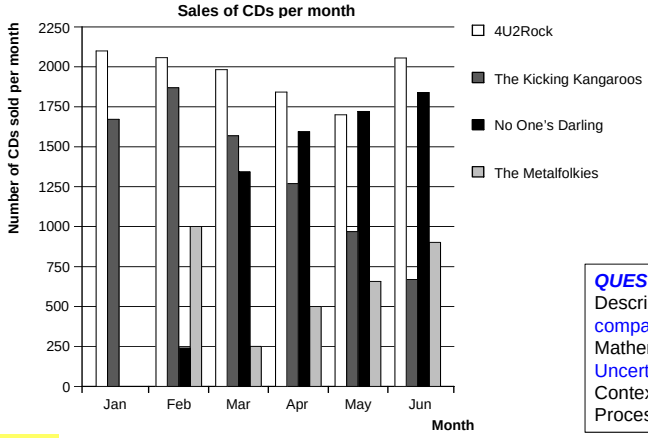
How many CDs did the band *The Metalfolkies* sell in April?
A. 250 B. 500 C. 1000 D.1270

QUESTION INTENT
Description: Read a bar chart
Mathematical content area: Uncertainty and data
Context: Societal
Process: Interpret

Full Credit (Below Level 1)
Code 1: B. 500

CHARTS

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



Q2

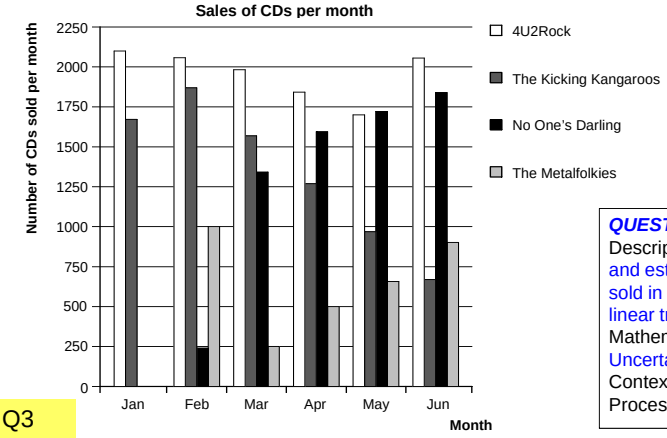
In which month did the band *No One's Darling* sell more CDs than the band *The Kicking Kangaroos* for the first time?
A. No month B. March C. April D. May

QUESTION INTENT
Description: Read a bar chart and compare the height of two bars
Mathematical content area: Uncertainty and data
Context: Societal
Process: Interpret

Full Credit (Level 1)
Code 1: C. April

CHARTS

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



QUESTION INTENT
Description: Interpret a bar chart and estimate the number of CDs sold in the future assuming that the linear trend continues
Mathematical content area: Uncertainty and data
Context: Societal
Process: Employ

Q3

The manager of *The Kicking Kangaroos* is worried because the number of their CDs that sold decreased from February to June.
What is the estimate of their sales volume for July if the same negative trend continues?
A. 70 CDs B. 370 CDs C. 670 CDs D. 1340 CDs

Full Credit (Level 2)
Code 1: B. 370 CDs

DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, D, in drops per minute for infusions.

They use the formula $D = \frac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

v is the volume in mL of the infusion

n is the number of hours the infusion is required to run.

Q1

A nurse wants to double the time an infusion runs for.
Describe precisely how *D* changes if *n* is **doubled** but *d* and *v* do not change.

Full Credit (Level 5)
Code 2: Explanation describes both the direction of the effect and its size.
- It halves
- It is half
- D will be 50% smaller
- D will be half as big

Partial Credit (Level 5)
Code 1: A response which correctly states EITHER the direction OR the size of the effect, but not BOTH.
- D gets smaller *[no size]*
- There's a 50% change *[no direction]*
- D gets bigger by 50%. *[incorrect direction but correct size]*

QUESTION INTENT:
Description: Explain the effect that doubling one variable in a formula has on the resulting value if other variables are held constant
Mathematical content area: Change and relationships
Context: Occupational
Process: Employ

Q2

Nurses also need to calculate the volume of the infusion, *v*, from the drip rate, *D*.

An infusion with a drip rate of 50 drops per minute has to be given to a patient for 3 hours. For this infusion the drop factor is 25 drops per millilitre.

What is the volume in mL of the infusion?

Full Credit (Level 5)
Code 1: 360 or a correctly transposed and substituted solution.
- 360
- $(60 \times 3 \times 50) \div 25$ *[Correct transposition and substitution.]*

QUESTION INTENT:
Description: Transpose an equation and substitute two given values
Mathematical content area: Change and relationships
Context: Occupational
Process: Employ