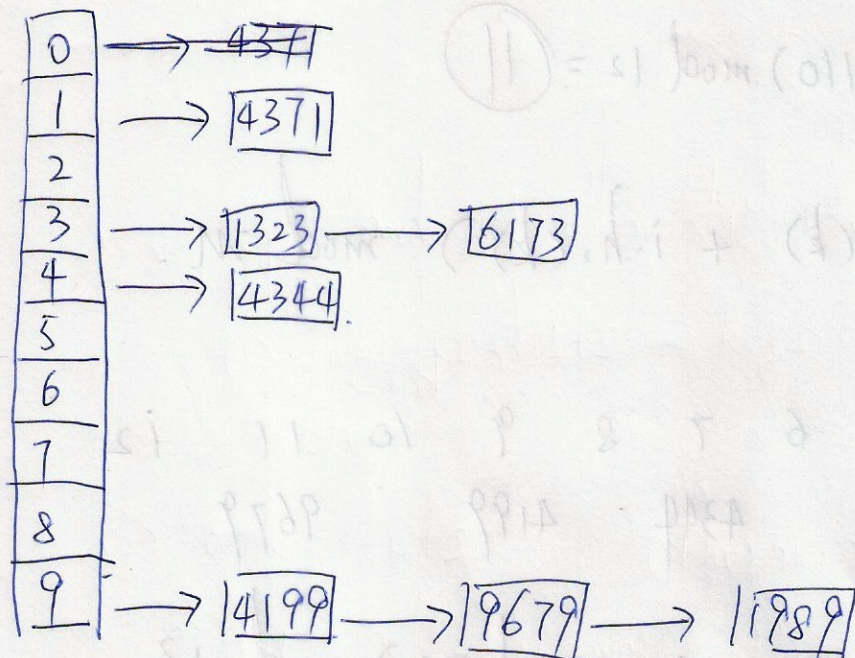


Ex 4.1

(1).



(2). suppose the slots range from 0 to 12 (you can change 12 to any value)

0	1	2	3	4	5	6	7	8	9	10	11
	4371		1323		6173	4344			4199	9679	1989

(3). suppose quadratic hash. $h(k, i) = (h(k) + i + i^2) \pmod{m}$

here $h(k) = k \pmod{10}$, $m = \cancel{10} 12$.

0	1	2	3	4	5	6	7	8	9	10	11
	4371		1323	4344	6173			4199	1989	9679	

$$h(1989, 0) = 9$$

$$h(1989, 5) = (9 + 5 + 25) \pmod{12} = 3$$

$$h(1989, 1) = 11$$

$$h(1989, 6) = (9 + 6 + 36) \pmod{12} = 3$$

$$h(1989, 2) = 3$$

$$h(1989, 7) = (9 + 7 + 49) \pmod{12} = 5$$

$$h(1989, 3) = 9$$

$$h(1989, 4) = (9 + 4 + 16) \pmod{12} = 5$$

$$h(1989, 8) = (9 + 8 + 64) \pmod{12} = 9$$

$$h(1989, 9) = (9 + 90) \bmod 12 = 3.$$

$$h(1989, 10) = (9 + 110) \bmod 12 = \textcircled{11}$$

$$(4). \quad h(k, i) = (h_1(k) + i \cdot h_2(k)) \bmod m.$$

$$m = 13.$$

0 1 2 3 4 5 6 7 8 9 10 11 12

4371 1989 1323 6173

4344

4199

9679

$$\begin{aligned} h(6173, 1) &= (3 + 7 - (6173 \bmod 7)) \bmod 13 \\ &= (10 - 6) \bmod 13 = 4. \end{aligned}$$

$$\begin{aligned} h(4344, 1) &= (4 + 7 - (4344 \bmod 7)) \bmod 13 \\ &= 7 \end{aligned}$$

$$\begin{aligned} h(9679, 1) &= (9 + 7 - (9679 \bmod 7)) \bmod 13 \\ &= 11 \end{aligned}$$

$$\begin{aligned} h(1989, 1) &= (9 + 7 - (1989 \bmod 7)) \bmod 13 \\ &= 15 \bmod 13 = 2. \end{aligned}$$

$$\begin{aligned} h(1989, 2) &= (\cancel{9 + 7 - (1989 \bmod 7)} \times 2) \bmod 13 \\ &= \cancel{15} \end{aligned}$$

(5) ~~refer~~ refer in Lecture Note.

Ex 4.14.

(1) unordered lists.

$O(N^2)$ when all keys have the same hash value, and not allow duplicate keys. otherwise $O(N)$.

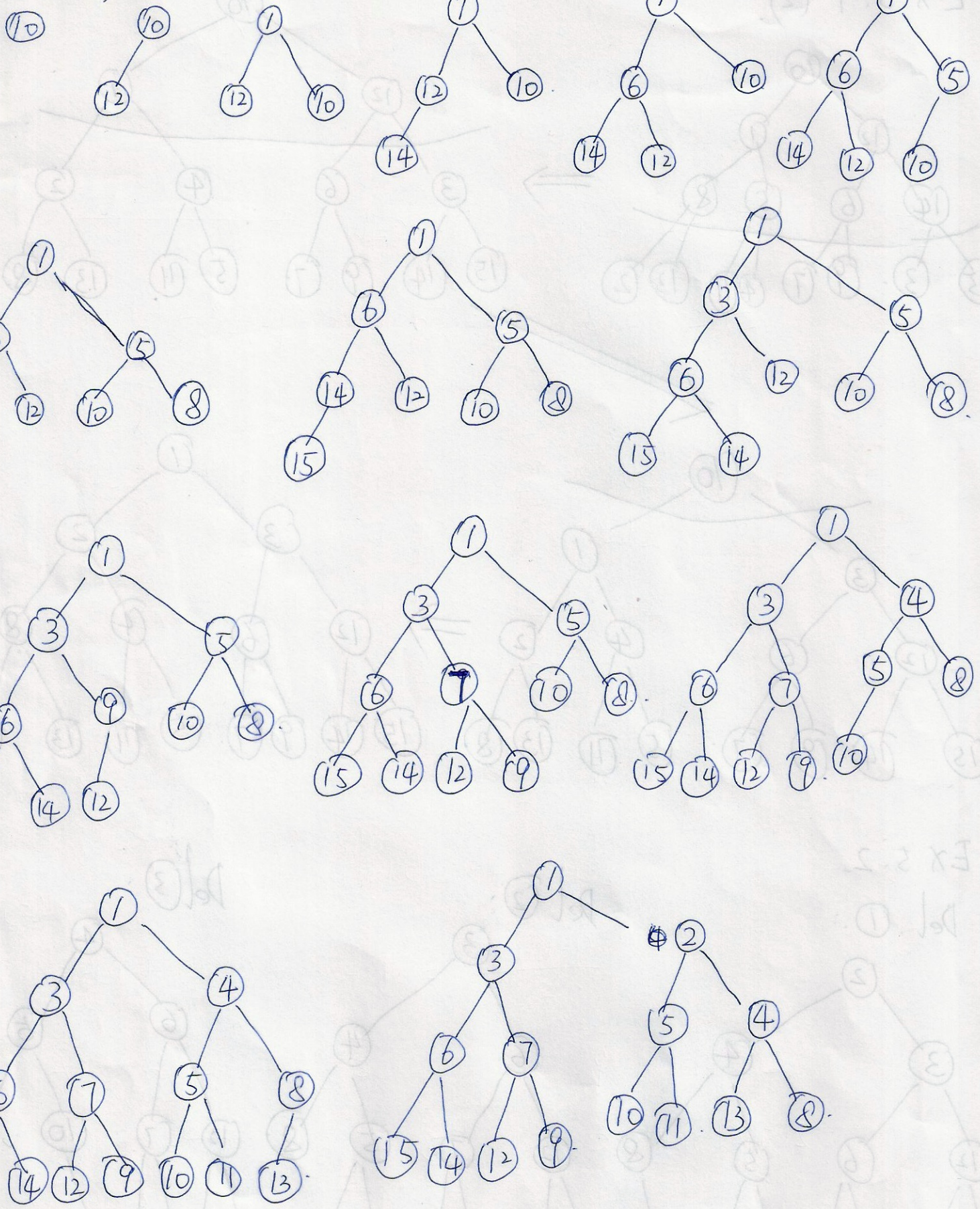
(2) order lists.

with self-balancing tree. worst case time is $O(N \log N)$. otherwise $O(N^2)$.

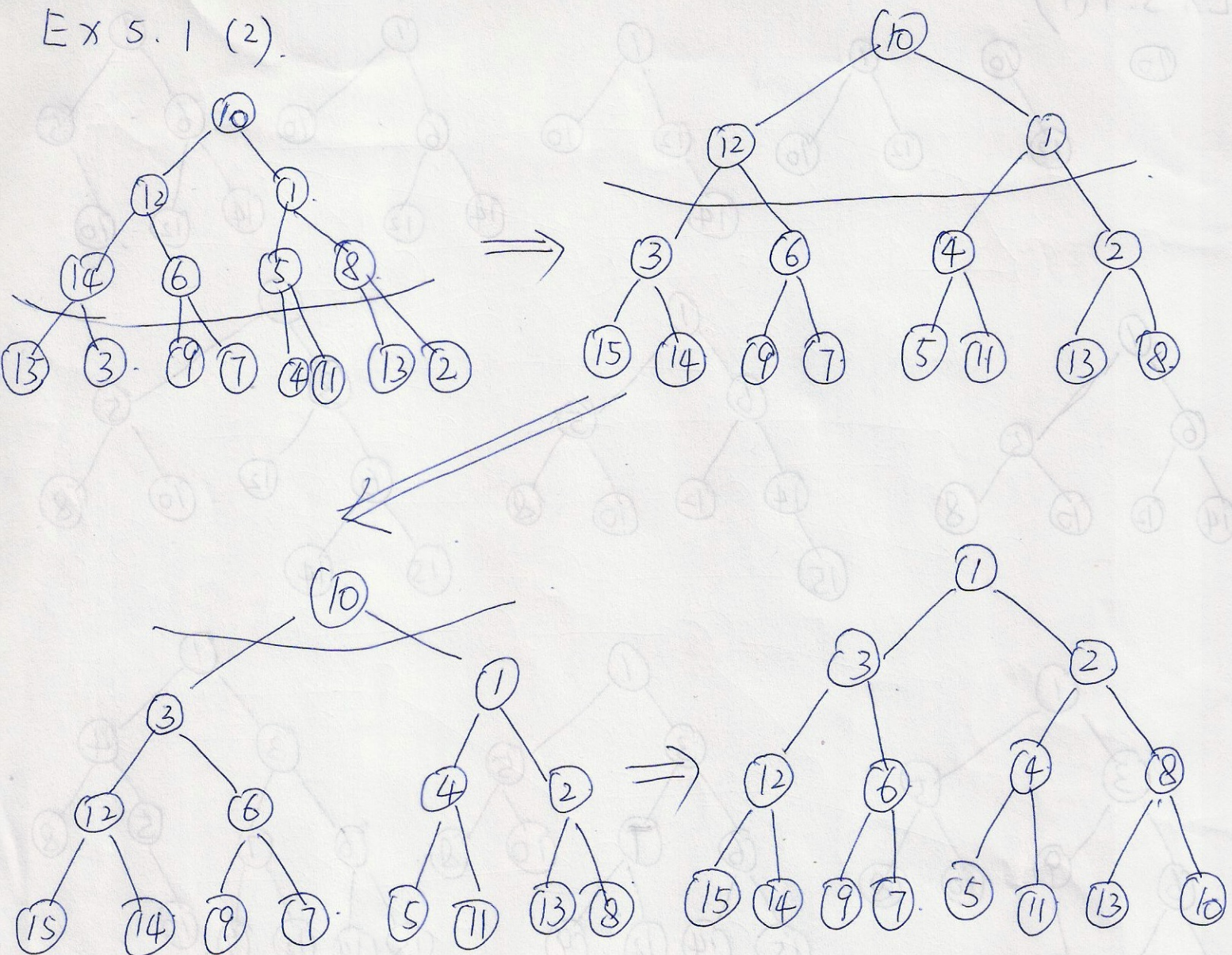
Ex 4.15.

~~$O(N^2)$~~ $O(N^2)$ when all keys have the same hash value.

Ex 5.1(1).

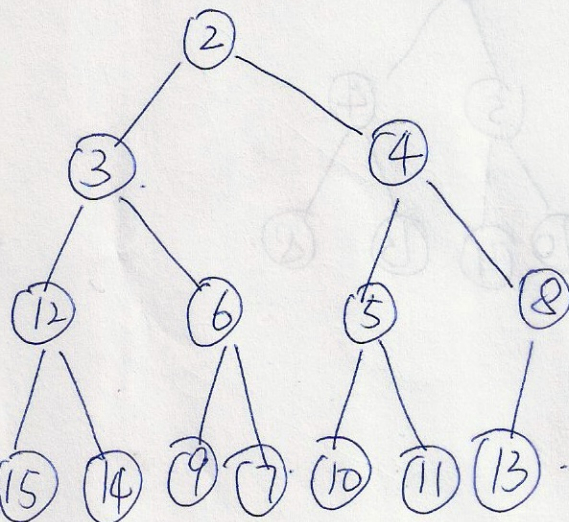


Ex 5.1 (2).

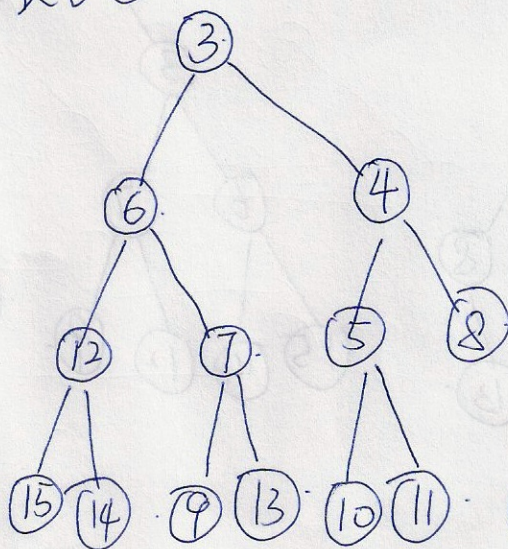


Ex 5.2.

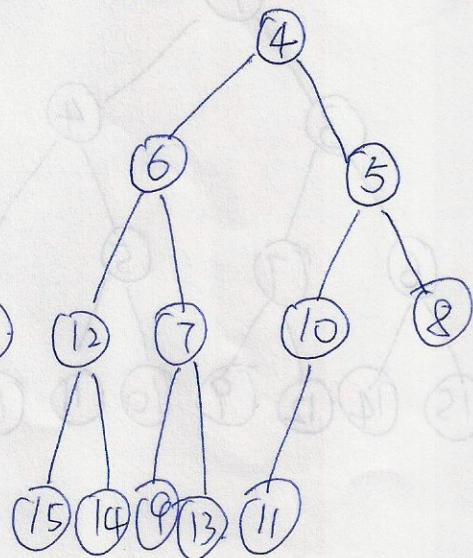
Del ①:



Del ②:



Del ③:



Ex 6.1. (1).

142 543 123 65 453 879 572.

434 111 242 811 102.

142: 123, 65, 111, 102.

543: 125, 65, 453, 434, 111, 242, 102.

123: 65, 111, 102.

453: 434, 111, 242, 102.

879: 572, 434, 111, 242, 811, 102.

572: 434, 111, 242, 102.

434: 111, 242, 102.

111: 102.

242: 102.

811: 102.

there are 34 pairs in total.

(2).

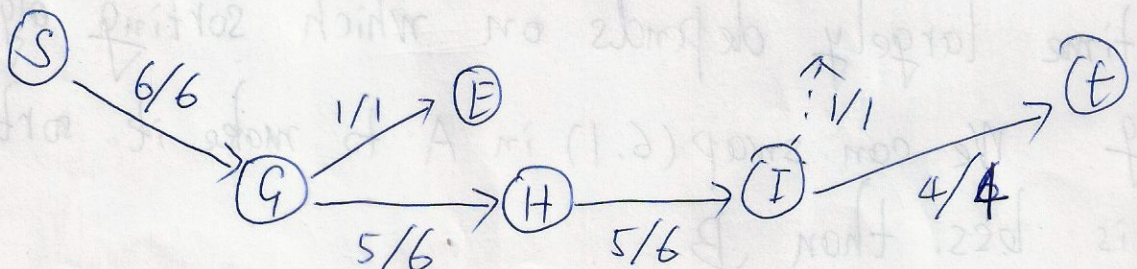
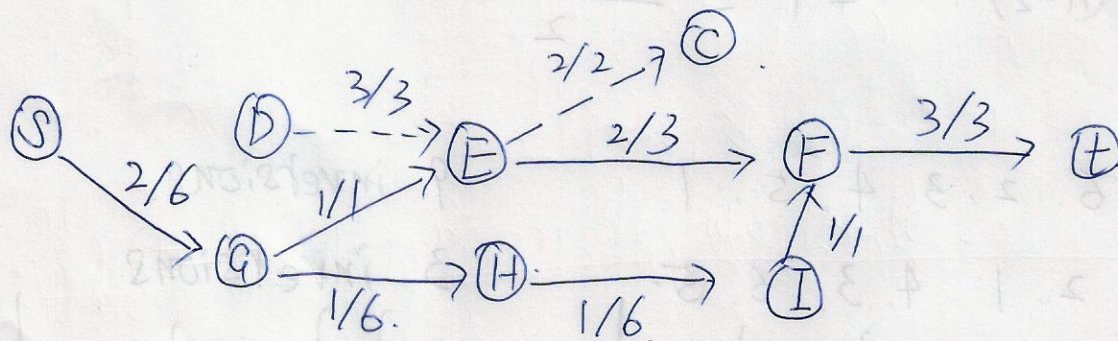
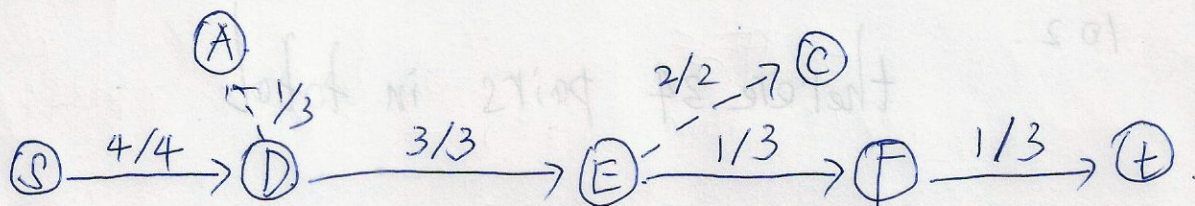
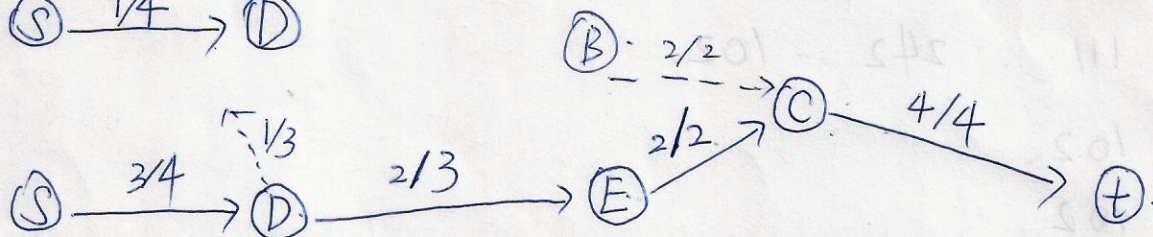
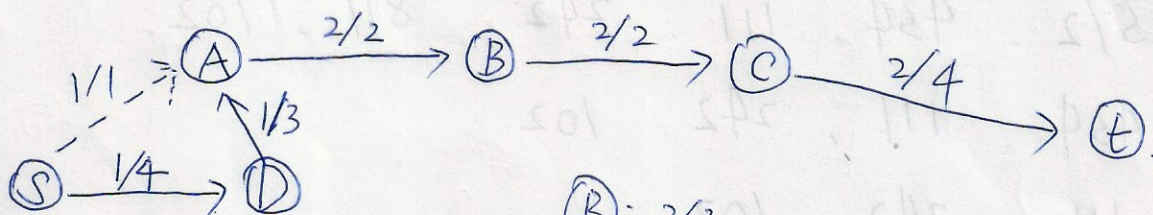
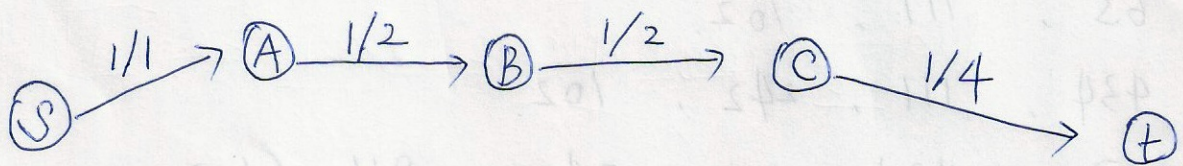
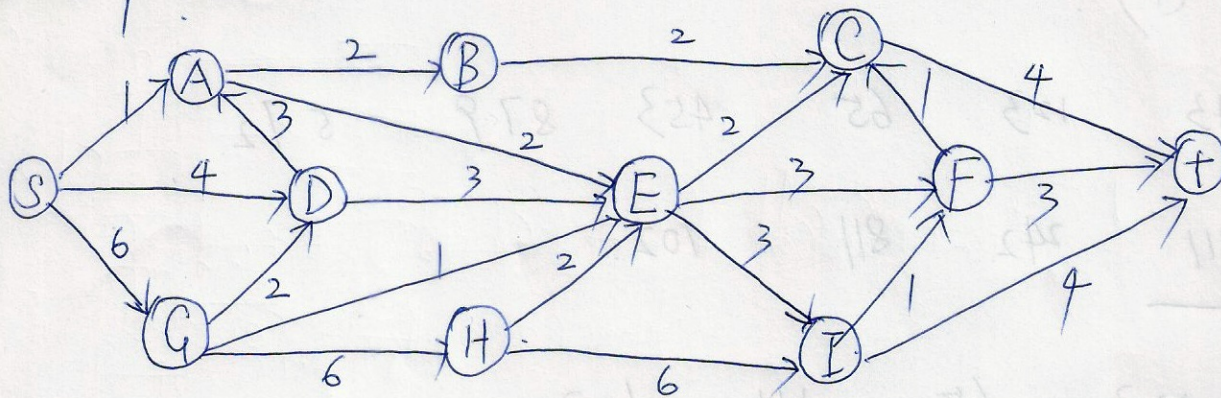
$$(n-1) + (n-2) + \dots + 1 = \frac{n(n-1)}{2}.$$

(3). A: 6, 2, 3, 4, 5, 1. 9 inversions.

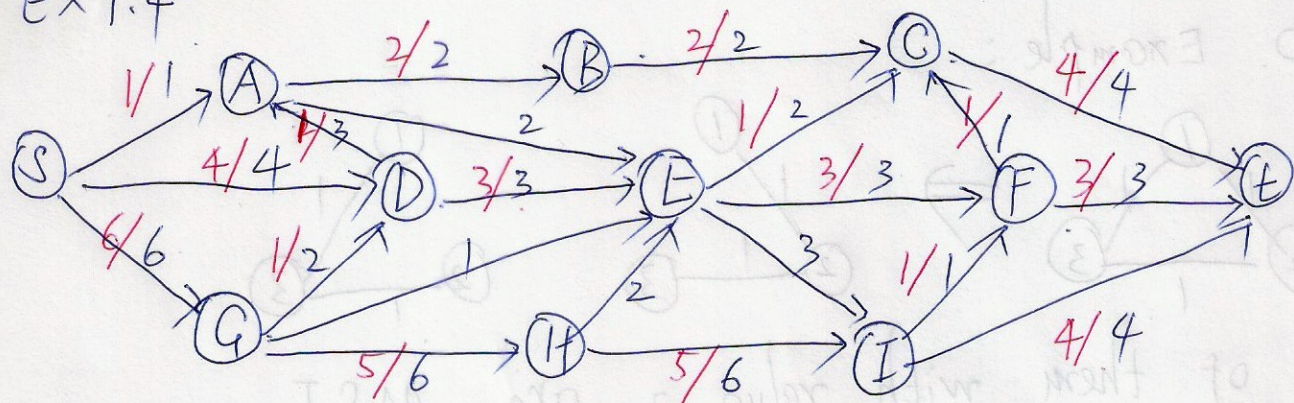
B: 2, 1, 4, 3, 6, 5. 3 inversions

running time largely depends on which sorting algorithm you're using. We can swap (6, 1) in A to make it sorted. Which is less than B.

Ex 7.4



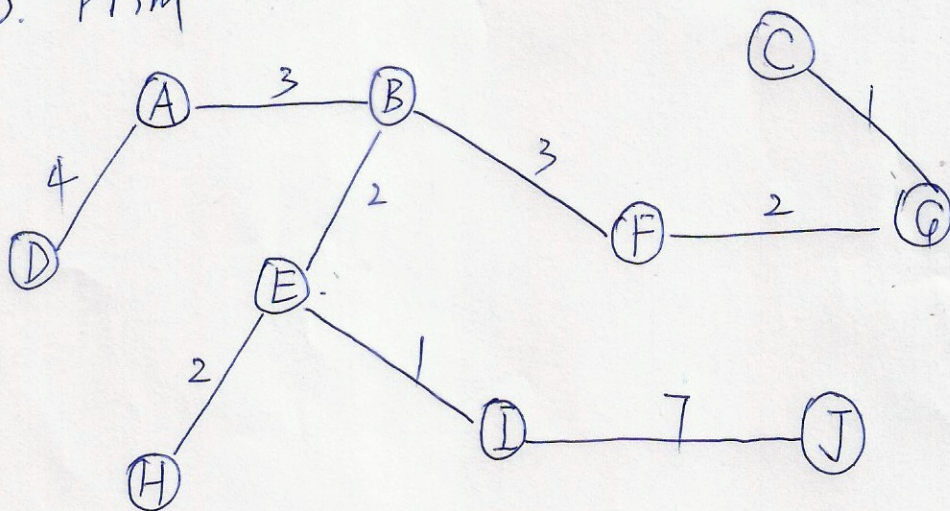
Ex 7.4



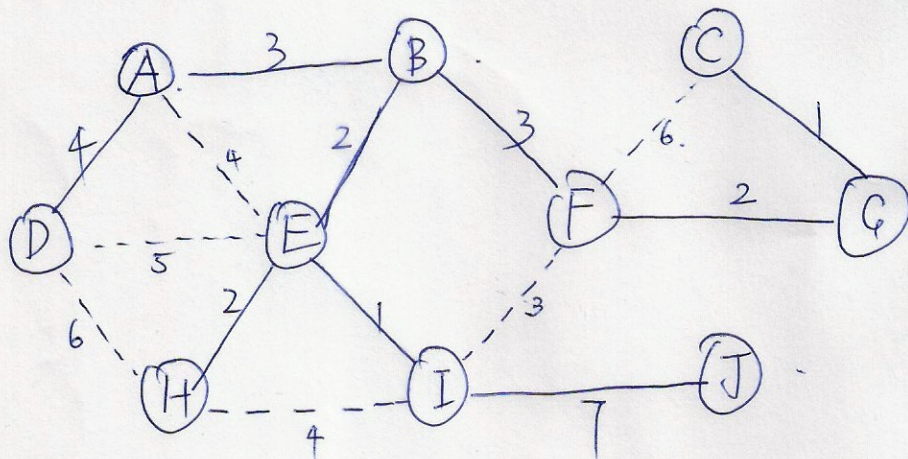
the max-flow is. $4+3+4 = 11$

Ex 7.5

(1). Prim



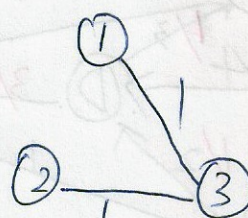
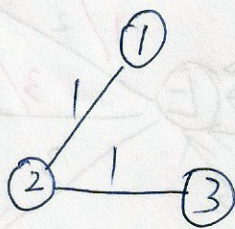
Kruskal.



edge to consider in $\mathcal{A}$.

(C,G), (E,I)
 (B,E) (F,G) (E,H)
 (A,B) (B,F) (F,I)
 (H,I) (A,D) (AE)
 (D,E)
 (D,H) (CF)
 (I,J).

(2.) NO. Example :



Both of them with value 2 are MST.