NO.

Date

1.6.(1) $f(n) = \sum_{i=1}^{n}\sum_{j=1}^{n} 1 = \sum_{i=1}^{n} n = n \cdot n = n^2$

$f(n) = O(n^2)$.

(2) ~~$f(n) = \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=1}^{i} 1 = \sum_{i=1}^{n}\sum_{j=i}^{n} j$~~

$f(n) = \sum_{i=1}^{n}\sum_{j=i}^{n} 1 = \sum_{i=1}^{n}(n-i+1) = \sum_{i=1}^{n} n - \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 = n^2 - \frac{n(n+1)}{2} + n = \frac{n(n+1)}{2} = \frac{1}{2}(n^2+n)$

$f(n) = O(n^2)$

(6). $f(n) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{j} 1 = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} j = \sum_{i=1}^{n-1} \frac{(i+1+n)(n-(i+1)+1)}{2} = \sum_{i=1}^{n-1} \frac{n^2+n-i^2-i}{2} = \frac{1}{2}[(n-1)(n^2+n) - \sum_{i=1}^{n-1} i^2 - \sum_{i=1}^{n-1} i]$

$= \frac{1}{2}(n^3-n - \frac{2(n-1)^3+3(n-1)^2+n-1}{6} - \frac{n(n-1)}{2}) = \frac{1}{3}(n^3-n) = \frac{n^3-n}{3}$

$f(n) = O(n^3)$

1.8(3)

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poly=0;                          *1
for(i=n; i>=0; i--)              *2
        poly= x*poly+ai;         *3
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The first line is an assignment, ~~cos~~ which costs 1 unit time.

The second line is the for-next loop set-up, which costs 2.3 unit time.

The third line costs $1+1.25+1.75=4$ time units, for there is an assignment, an addition and a multiplication. ~~We also~~ We ~~also~~ have to add another 1.5 time units for each loop, $4+1.5=5.5$.

And i from n to 0 is $n+1$ times.

~~for tot~~ In total, $5.5(n+1)+1+2.3 = 5.5n+8.8$ units.

In big O notation, $5.5n+8.8$ is $O(n)$.

1.9(2)

Algorithm A: $t(10) = 10$ $\quad S(10) = 10$

$t(20) = 20$ $\quad S(20) = 1.5 \times 20 = 30$

$t(30) = 30$ $\quad S(30) = 1.5 \times 30 = 45$

$t(50) = 50^3 = 125000$ $\quad S(50) = 1.5 \times 50 = 75$

$t(70) = 70^3 = 343000$ $\quad S(70) = 1.5 \times 70 = 105$

$t(100) = 1000000$ $\quad S(100) = 150$

Algorithm B: $t(10) = 10$ $\quad S(10) = 5 \times 10 = 50$

$t(20) = 20$ $\quad S(20) = 5 \times 20 = 100$

$t(30) = 30^2 = 900$ $\quad S(30) = 5 \times 30 = 150$

$t(50) = 50^2 = 2500$ $\quad S(50) = 0.5 \times 50 = 25$

$t(70) = 70^3 = 343000$ $\quad S(70) = 0.5 \times 70 = 35$

$t(100) = 1000000$ $\quad S(100) = 50$

(3) ~~C(n)~~ ~~Average is~~ Sum is, $\sum_{n=1}^{100} C(n) = \sum_{n=1}^{100} t(n) + 5 \cdot \sum_{n=1}^{100} S(n)$

Sum of $C_A(n)$: $\sum_{i=1}^{100} C_A(i) = \sum_{i=1}^{9} (i^2+5i) + \sum_{i=10}^{19} (i+5i) + \sum_{i=20}^{49} (i+7.5i) + \sum_{i=50}^{100} (i^3+7.5i)$

$= 24040740.$

Average is ~~CA~~ $24040740/100 = 240407.4.$

Sum of $C_B(n)$: $\sum_{i=1}^{100} C_B(i) = \sum_{i=1}^{29} (i+2.5i) + \sum_{i=30}^{49} (i^2+2.5i) + \sum_{i=50}^{69} (i^2+2.5i) + \sum_{i=70}^{100} (i^3+2.5i)$

$= 19814237.5$

Average is $19814237.5/100 = 198142.375.$

Average of A > Average of B. Thus total cost of B is smaller, B is better.

~~Average of~~ (Average cost of A − Average cost of B)/Average ~~of~~ cost of A

$= 0.17581 = 17.581\%$. Better by 17.581%