語言及現代學語言系 Linguistics and Modern Languages

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SID:

[kɔ:s]:

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1.1)1) $\sum_{i=0}^{\infty} \frac{1}{4^i}$

$= \frac{1}{4^0} + \frac{1}{4^1} + \frac{1}{4^2} + \dots + \frac{1}{4^n} = S$

$4S = 4 + 1 + \frac{1}{4^1} + \frac{1}{4^2} + \dots + \frac{1}{4^{n-1}}$

$4S - S = 4 + \frac{1}{4^n}$ $(\because n$ tends to $\infty$, $\frac{1}{4^n} = 0)$

$3S = 4$

$S = 4/3$ //

9) $\sum_{i=0}^{n} n^4 - (n-1)^4 = n^4 - (n-1)^4 + (n-1)^4 - (n-2)^4 + \dots + 1^4 - 0^4 + 0^4 - (-1)^4$

$= n^4 \cancel{0} - 1$

$n^4 - (n-1)^4 = n^4 - (n^4 - 4n^3 + 6n^2 - 4n + 1)$

$= 4n^3 - 6n^2 + 4n - 1$

$\sum_{i=0}^{n} n^4 - (n-1)^4 = \sum_{i=0}^{n} 4n^3 - \sum_{i=0}^{n} 6n^2 + \sum_{i=0}^{n} 4n - \sum_{i=0}^{n} 1$

$= 4\sum_{i=0}^{n} n^3 - 6\sum_{i=0}^{n} n^2 + 4\sum_{i=0}^{n} n - n - 1 = n^4 - 1$

$4\sum_{i=0}^{n} n^3 = n^4 + 6\sum_{i=0}^{n} n^2 - 4\sum_{i=0}^{n} n + n$

$= 6[\frac{n(n+1)(2n+1)}{6}] - 4[\frac{n(n+1)}{2}] + n + n^4$

$= n(n+1)(2n+1) - 2n(n+1) + n(n^3 + 1)$

$= n(2n^2 + 3n + 1 - 2n - 2 + n^3 + 1)$

$= n(n^3 + 2n^2 + n)$

$= n^2(n^2 + 2n + 1)$

$= n^2(n+1)^2$

$\sum_{i=0}^{n} n^3 = n^2(n+1)^2 / 4$

$= [\frac{n(n+1)}{2}]^2$ //

3) Let $k$ be the constant that $2^n \geq 2^{2n}$

$$2^{2n} \leq k2^n$$
$$2^{2n-n} \leq k$$
$$2^n \leq k$$

As $n$ is tends to infinite,
$\therefore$ There is no $k$ larger than $2^n$
$\therefore 2^{2n} \neq O(2^n)$ //

1.3) 4) $T(n) = aT(n/2) + bn^c$

$$= a[aT(n/2^2) + b(n/2)^c] + bn^c$$
$$= a^2 T(n/2^2) + ab\frac{n^c}{2^c} + bn^c$$
$$= a^2[aT(n/2^3) + b(n/2^2)^c] + ab\frac{n^c}{2^c} + bn^c$$
$$= a^3 T(n/2^4) + a^2 b(\frac{n}{2^2})^c + ab(\frac{n}{2})^c + bn^c$$
$$= a^{\log_2 n} T(1) + bn^c [1 + \frac{a}{2^c} + (\frac{a}{2^c})^2 + \dots + (\frac{a}{2^c})^{\log_2 n - 1}]$$
$$= a^{\log_2 n} + bn^c \left[\frac{1 - (\frac{a}{2^c})^{\log_2 n}}{1 - \frac{a}{2^c}}\right]$$ //

8) $T(n) = 3T(n-1) + 2$

$$= 3[3T(n-2) + 2] + 2$$
$$= 3^2 T(n-2) + 3 \cdot 2 + 2$$
$$= 3^2[3T(n-3) + 2] + 3 \cdot 2 + 2$$
$$= 3^3 T(n-3) + 3^2 \cdot 2 + 3 \cdot 2 + 2$$
$$= 3^{n-1} T(1) + 2(1 + 3 + 3^2 + \dots + 3^{n-2})$$
$$= 3^{n-1} + 2\left(\frac{1 - 3^{n-1}}{1-3}\right) = 3^{n-1} - 1 + 3^{n-1} = 2 \cdot 3^{n-1} - 1$$ //

1.4) 2) Assume $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$,

Base case, $n = 0$: $2^0 = 1 = 2^{0+1} - 1$

$$L.H.S. = 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n + 2^{n+1}$$
$$= 2^{n+1} - 1 + 2^{n+1} \qquad = 2^{n+2} - 1$$
$$R.H.S. = 2^{n+1+1} - 1 \qquad = 2^{n+2} - 1 = L.H.S.$$

$\therefore \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

6) for $n = 17$

$17^2 + 17 + 17 = 323$

where $323/17 = 19$

$\therefore$ 323 is not prime number

$\therefore$ The formula's number is not prime.