Yonsei University Intensive Course
Gravitational Waves
Exercise Set 2

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I Linearized general relativity

(I.1) Define

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h. \tag{1}$$

Calculate how $\bar{h}_{\mu\nu}$ changes under gauge transformations. Then show that for any field configuration $h_{\mu\nu}(x)$, one can find a gauge transformation such that

$$\partial^\nu \bar{h}_{\mu\nu} = 0. \tag{2}$$

(Hint: first calculate how $\partial^\nu \bar{h}_{\mu\nu}$ changes under gauge transformations. Then, using the Green's function of the d'Alembertian, construct the gauge transformation which makes it zero.)

(I.2) Calculate the Riemann tensor $R_{\mu\nu\rho\sigma}$ in the linearized theory and show that its components are gauge invariant.

(I.3) Show that in the harmonic gauge the linearized Einstein equations are

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}. \tag{3}$$

II Gravitational Wave Propagation in a Viscous Fluid

A gravitational wave encounters a viscous fluid, which is initially at rest with fluid four-velocity given by $u^a = (1, 0, 0, 0)$.

(II.1) The shearing of the fluid is decribed by the shear tensor

$$\sigma_{\alpha\beta} = \frac{1}{2}\nabla_\alpha \mu_\beta + \frac{1}{2}\nabla_\beta u_\alpha + \frac{1}{2}u_\alpha u^\mu \nabla_\mu u_\beta + \frac{1}{2}u_\beta u^\mu \nabla_\mu u_\alpha - \frac{1}{3}(g_{\alpha\beta} + u_\alpha u_\beta)\nabla_\mu u^\mu \tag{4}$$

where ∇_μ is the covariant derivative. Show that the shear caused by gravitational wave in the TT-gauge is purely spatial with

$$\sigma_{ij} = \frac{1}{2}\frac{\partial}{\partial t}h^{TT}_{ij} \tag{5}$$

(II.2) The shearing of the viscous fluid generates a contribution to the stress-energy tensor of the form

$$T^{\text{visc}}_{\alpha\beta} = -2\eta\sigma_{\alpha\beta} \tag{6}$$

where η is the coefficient of viscosity. The linearized field equations for the gravitational wave are therefore

$$\Box h^{TT}_{ij} = \eta\frac{16\pi G}{c^4}\frac{\partial}{\partial t}h^{TT}_{ij} \tag{7}$$

Show that a plane wave travelling along the z-axis is attenuated by the fluid by an amount $e^{-x/l}$ where l is the attenuation length scale

$$l = \frac{c^3}{8\pi G\eta} \tag{8}$$

(II.3) Ketchup has a coefficient of viscosity of $\eta = 50kg/(ms)$. Calculate the distance l that a gravitational wave must travel through ketchup before it is attenuated by a factor of $1/e$.