Intensive Course in Physics
Gravitational Waves

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Chapter 4: Gravitational-wave Detectors

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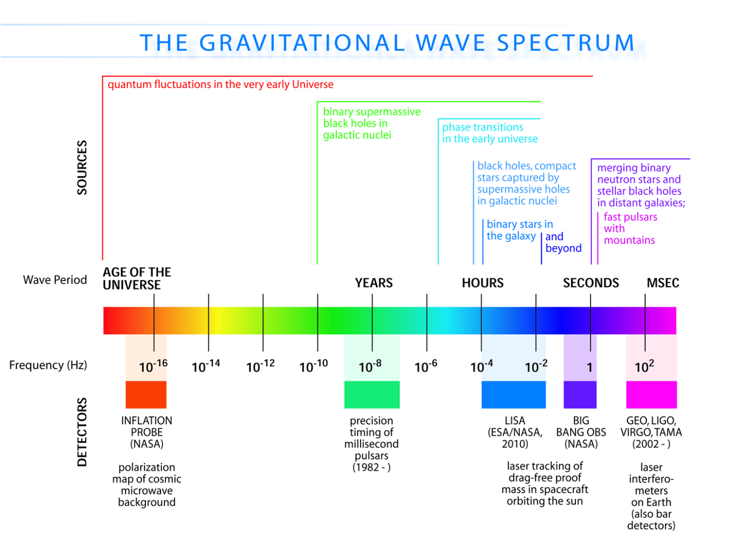
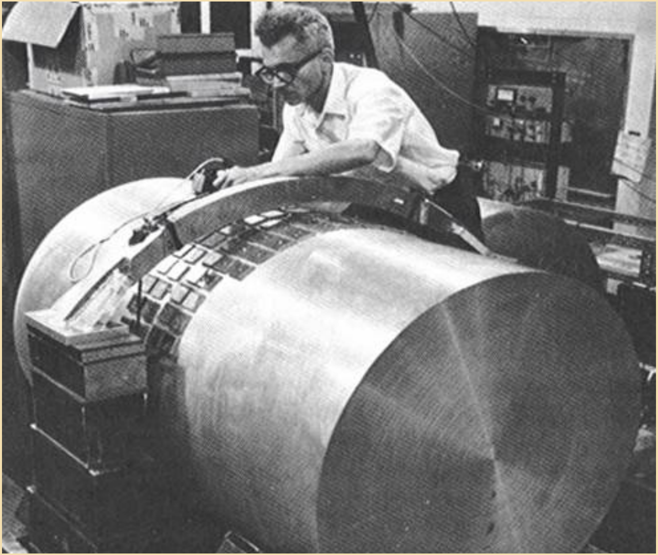
Gravitational-wave spectrum



Bar detectors



Response of Bar Detectors

- Assume 1-dimensional bar of lenght L and mass M described by

$$x = x_0 + u(t, x) \tag{1}$$

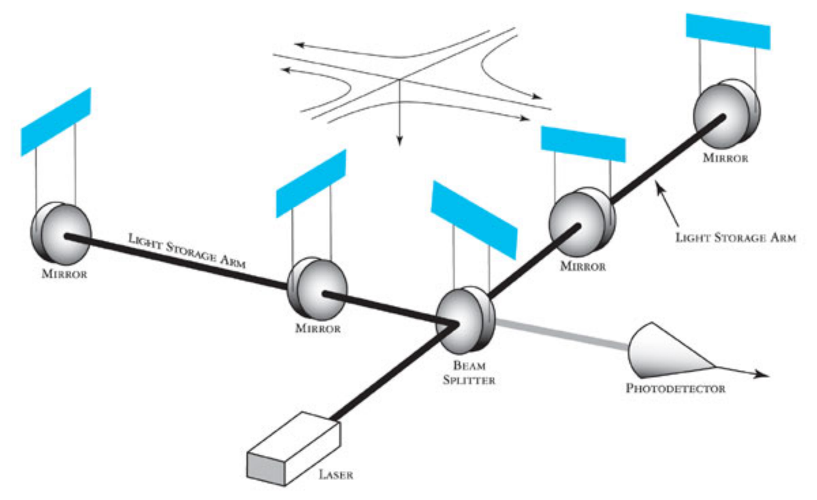
- For a short GW burst, we assume a elastic bar without dissipation

$$dm\left(\frac{\partial^2 u}{\partial t^2} - v_s^2 \frac{\partial^2 u}{\partial x^2}\right) = dFx(t, x) \tag{2}$$

- For boundary conditions $\partial u/\partial x(x = \pm L/2) = 0$
- The response is in terms of the signal energy E_s

$$\left|\tilde{h}_{xx}(f_0)\right| = \frac{1}{4Lf_0^2}\sqrt{\frac{E_s}{M}} \tag{3}$$

Interferometers



Response of Interferometers

- Interferometers with arms of L
- For the plus polarisation

$$ds^2 = -c^2 dt^2 + [1 + h_+(t)]\, dx^2 + [1 - h_+(t)]\, dy^2 + dz^2 \tag{4}$$

- Consider the Electric fields at the beamsplitter

$$E_{\text{tot}} = -\imath E_0 e^{-\imath\omega_L}(t - 2L/c) \sin(\phi_0 + \Delta\phi_x) \tag{5}$$

$$\Delta\phi_x(t) = h_0 \frac{\omega_L L}{c} \operatorname{sinc}\left(\frac{\omega_{\text{GW}} L_x}{c}\right) \cos[\omega_{\text{GW}}(t - L_x/c)] \tag{6}$$

Matched Filtering

- Use a filter to $K(t)$ to filter the signal $s(t)$

$$\hat{s} = \int_{\infty}^{\infty} dt s(t) K(t) \tag{7}$$

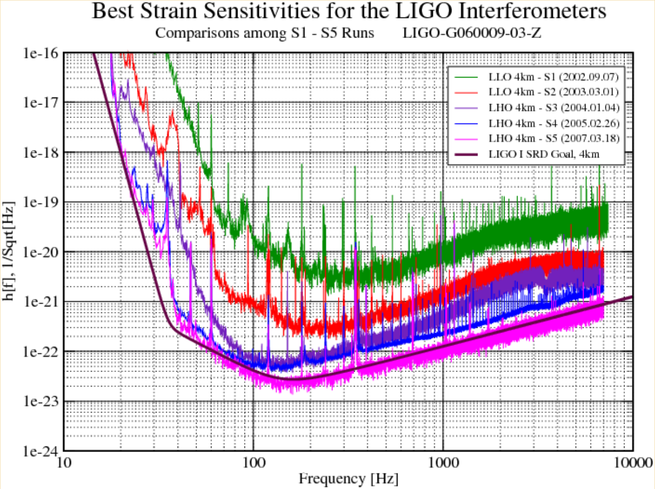
- Find the filter that maximises the signal-to-noise ratio

$$\tilde{K}(f) \propto \frac{\tilde{h}(f)}{S_n(f)} \tag{8}$$

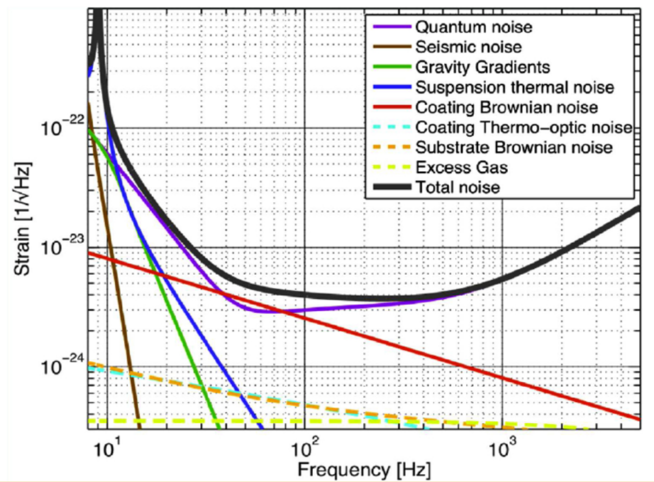
- where we have introduced the noise power-spectral densitiy

$$\left\langle \tilde{n}^*(f)\tilde{n}(f') \right\rangle = 1/2\delta(f - f')S_n(f) \tag{9}$$

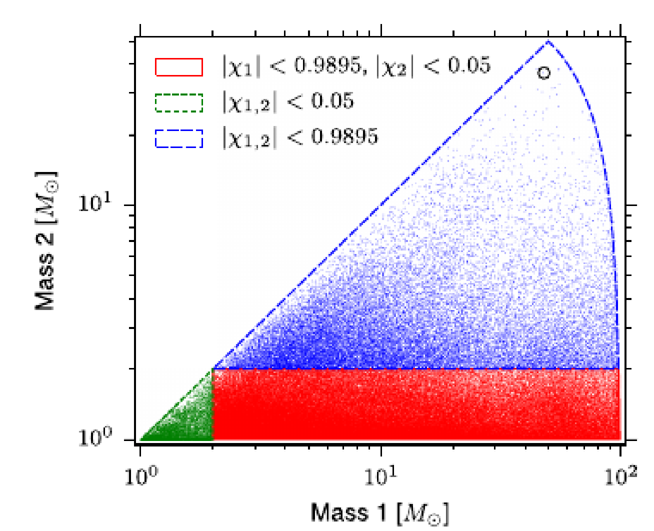
Power Spectral Density



Noise Sources



Template Bank



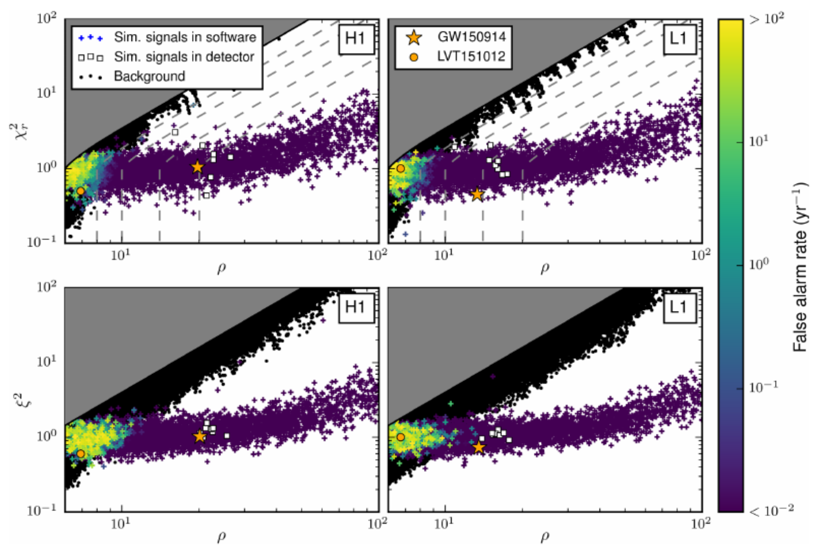
Signal-based discriminator

- Use a signal-based discriminator

$$\chi^2 = \frac{1}{p}\sum_{j=1}^{p}(z - pz_j)^2 \tag{10}$$

$$z_j = 4\Re\left\{\int_{f_{j-1}}^{f_j} \frac{\tilde{s}(f)\tilde{h}(f)}{S_n(f)}\right\} \tag{11}$$

Discriminating signal from noise



Detection statistic