

Course Examinations Second Term, 2012-2013**Course Code & Title** : **CSCI2100B Data Structures****Time allowed** :**1**..... **hours****40**..... **minutes****Student I.D. No.** : **Seat No. :**

You have 100 minutes on this 100 points examination. This means that you should spend about 1 minute for each 1 point. Read all questions carefully and answer all of them in the answer booklet provided. More booklets are available upon request.

1. (25) On Program Analysis

You are given the task to analyze two data structures, D_1 and D_2 , and their related algorithms to solve a problem with n input elements. Now, D_1 and D_2 have the following time complexity for three different functions, f_1 , f_2 , and f_3 with their Best, Average, and Worst time complexity respectively as follows:

Table 1: Time Complexity for D_1 and D_2

	D_1	D_2
f_1		
Best	$2n + 5$	10
Average	$100 \log n$	$3n$
Worst	$n^2 \log n$	$n \log^2 n$
f_2		
Best	$10 \log n$	$n \log n$
Average	$\log^2 n + 10n$	$\log n + n^2$
Worst	$n!$	$5n^7 + n^4 + 4n^2 + 2$
f_3		
Best	$\log n$	20
Average	$15n$	$n^{1.5}$
Worst	$5n \log n$	$n^2 + 5(\log n)^2$

(1)

- (a) (9) Draw another table of the same size and same format but fill each entry with the corresponding time complexity in the Big-O notation from Table 1.

- (b) (4) Estimate for what n is the interception point for the Average case of f_1 for both D_1 and D_2 . In other words, where is the n where $f_1^{D_1}(n-1) \geq f_1^{D_2}(n-1)$ and $f_1^{D_1}(n) \leq f_1^{D_2}(n)$ for the Average time complexity.
- (c) (6) Use only the Average Big-O time complexity to order f_1 , f_2 , and f_3 for D_1 and D_2 respectively. For instance, for D_1 , $f_3 \leq f_1 \leq f_2$. (hint: you should have one for D_1 and one for D_2).
- (d) (6) Which one of the three function for each D_1 and D_2 is the most crucial in terms of the affect on time complexity in the Worst case? Justify your answer.

2. (14) On Stacks and Queues

- (a) (6) Given the same initial sequence of 1, 2, 3, 4, 5, 6, 7, your target is to judge whether this sequence can be changed to another sequence (final sequence) using a stack or a queue. Please fill the blanks in the following table with **Yes** or **No**.

<i>Final Sequence</i>	<i>Stack</i>	<i>Queue</i>
1, 3, 2, 5, 4, 7, 6		
4, 1, 2, 3, 7, 5, 6		
3, 6, 7, 5, 4, 2, 1		

(2)

- (b) (8) In (a), How many different possible final sequences can be generated using a stack? How about using a queue?

3. (15) On Trees

- (a) (5) Draw a complete binary tree with height = 2. Using Post-order traversal, label all the nodes in the order of traversal with unique integers, 1, 2, ... (so the traversal will result in the unique integer sequence).
- (b) (6) Using the labelled tree above, draw two of the three different implementations for the tree as discussed in the class. Label clearly the details of the implementation for each drawing.
- (c) (2) In (a), if we change the height to n , what is the label of the root of the complete binary tree.
- (d) (2) In (c), if we further change the Post-order to In-order, what is the label of the root of the complete binary tree.

4. (20) On AVL Trees

- (a) (10) Show the result of inserting 30, 20, 15, 25, 28, 22, 35, 27 into an empty AVL tree. Show ALL intermediate steps and indicate whether no rotation, a single rotation, or a double rotation has been performed in each step.
- (b) (2) What is the height of the resulting AVL tree?
- (c) (2) What is the depth of the node with the label "4" in the above resulting AVL tree?
- (d) (2) List out all the leaf nodes in the above resulting AVL tree.
- (e) (4) What is the minimum deletion sequence of the nodes that will result in a double rotation of the above AVL tree? Show the result after the deletion sequence has been performed. Label each step clearly.

5. (26) Short Answers (Please give only concise and short answers!)

- (a) (3) If a language in which the operators are associated right-to-left, i.e., $x * y * z = x * (y * z)$, what is the value of the following expression: $10 - (20 * 8 / (5 - 2 - 1)) - 20$?
- (b) (2) Show that $(n + 1)^2 = O(n^2)$. Justify your answer.
- (c) (3)

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if  $A > B$  then
     $x = F(y)$ 
else if  $B > C$  then
     $x = G(z)$ 
end if

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Now, if the expressions " $A > B$ " and " $B > C$ " are independent and that on the average, $A > B$, 70% of the time and $B > C$, 30% of the time. If this program segment above is executed 100,000 times, how many times would one expect the function $F()$ and $G()$ to be executed?

- (d) (3) If all variables are of type *integer* and if $a \equiv b \pmod{m}$ and $x \equiv y \pmod{m}$, which of the following (A) to (E) must be true?
- $a + x \equiv b + y \pmod{m}$
 - $ax \equiv by \pmod{m}$
 - $a/n \equiv b/n \pmod{m}$ for all $n \neq 0$.
- (A) ii only
 (B) iii only
 (C) i and ii only
 (D) i and iii only
 (E) i, ii, and iii
- (e) (2) Rewrite the following expression in the infix order to the postfix order: $((F + E) / (A * B)) + (C * D)$.
- (f) (4) Show at least 2 different implementations of a queue data structure. Illustrate at least one advantage for each implementation.
- (g) (2) What is the major difference between the binary search tree and binary trees?
- (h) (2) List two operations possible for general lists that are not allowed for either stacks or queues.
- (i) (2) Name two operations that doubly linked list is better than the simply linked list.
- (j) (3) Solve $T(n) = aT(n - 1) + bn, T(1) = 1$.

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