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THE CHINESE UNIVERSITY OF HONG KONG

Department of Statistics

will present a seminar entitled

Estimation of the Hurst index on finite frequency bands. Applications in health

by

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on

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in

Lady Shaw Building LT1
The Chinese University of Hong Kong

Estimation of the Hurst index on finite frequency bands. Applications in health

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We deal with Gaussian processes $\{X(t), t \in R\}$ having zero mean and stationary increments. These processes can also be written using the following harmonizable representations, see [3]:

$$X(t) = \int_{\mathbb{R}} \left(e^{it\xi} - 1\right) f^{1/2}(\xi)\, dW(\xi), \quad \text{for all } \ t \in R, \tag{1}$$

where $W(dx)$ is a complex Brownian measure, with adapted real and imaginary part such that the Wiener integral is real valued and

- f is a positive even function, called the spectral density of X, such that

$$\int_R \left(1 \wedge |\xi|^2\right) f(\xi)\, d\xi < \infty. \tag{2}$$

When X is a fractional Brownian motion (fBm) the spectral density follows a power law $f(\xi) = \sigma^2 |\xi|^\beta$ with $\beta = 2H + 1$ where H denote the Hurst index or in log-log scale $\ln f(\xi) = \beta \ln|\xi| + \ln \sigma^2$ for all the frequencies from zero to the infinity.

However, for certain applications in medicine or health (heartbeat time series or human posture for instance) the behavior of the spectral density differs according to scales and this brings relevant biological information,s see below the example of heart rhythm: We propose a

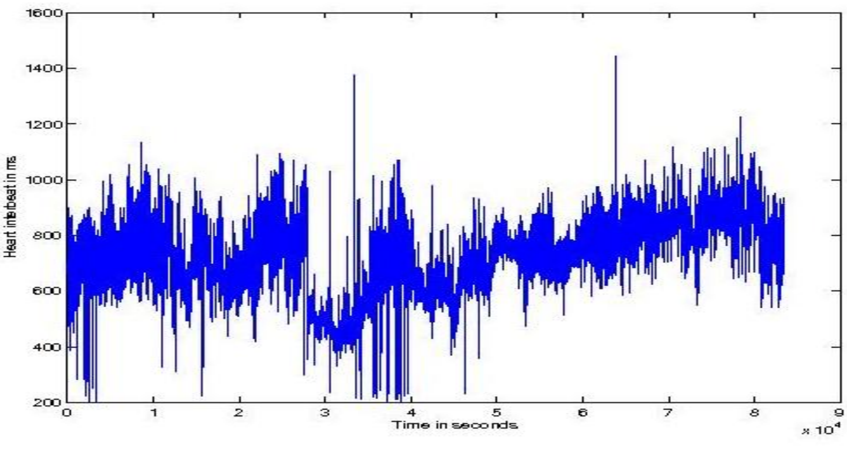

Figure 1: *RR interval for a healthy subject during a period of 24 hours .*

statistical study of the estimation of the spectral density f on one or several finished frequency bands (ω_k, ω_{k+1}) with for eg. $k = 1$ or 2. We limit ourselves to the case of processes, but we think that this approach could be adapted in superior dimension.

Two cases of observation are considered: first, an idealistic one, where a continuous time path of the process is known, second, a more realistic one, of observation of the process at random times.

By using wavelet analysis, one derives a non parametrical estimator of the spectral density function $\xi \mapsto f(\xi)$. One gives Central Limit Theorems (CLT) and estimations of the Mean Integrated Square Error (MISE) in both cases of observation.

Next, one presents numerical experiments and applications to heartbeat time series: one estimates the spectral density on the frequencies bands $(0.04\,Hz,\ 0.15\,Hz)$ and $(0.15\,Hz,\ 0.5\,Hz)$ corresponding respectively to the orthosympathic and the parasympathic nervous systems following [5]. These results are based on real data for healthy subject during a period of 24 hours or on marathon runners during 3 hours.

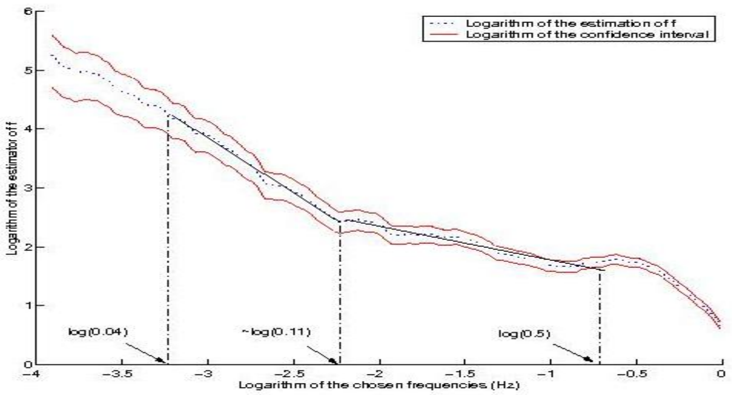

Figure 2: *spectral density estimation during the working hours.*

References

[1] Bardet, J.M. & Bertrand, P. R. (2010), "Nonparametric estimation of the spectral density of a continuous-time Gaussian process observed at random times", to appear in *Scand. J. of Stat.*

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[3] Cramér, H. and Leadbetter, M. R. (1967). *Stationary and related stochastic processes. Sample function properties and their applications.* Wiley and Sons.

[4] Lii, K.S. and Masry, E. (1994) Spectral estimation of continuous-time stationary processes from random sampling. *Stochastic Process. Appl.* **52**, no. 1, 39–64.

[5] Task force of the European Soc. Cardiology and the North American Society of Pacing and Electrophysiology (1996), Heart rate variability. Standards of measurement, physiological interpretation, and clinical use. *Circulation* 93 (5), 1043-1065.